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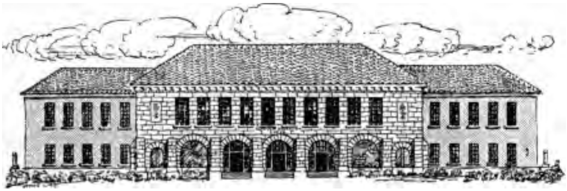

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THE FOUNDATIONS

OF

HIGHER ARITHMETIC

BY

B. F. SISK, M.S., M.A.

SUPERINTENDENT OF SCHOOLS, ROSEBUD, TEXAS, AND AUTHOR OF
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"GRAMMAR AS A SCIENCE"

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PREFACE

This work is an outgrowth of the author's experience in teaching arithmetic to advanced classes in high schools and in normals, and has been prepared especially to meet the needs of such classes. In most of the high schools, pupils are required to review arithmetic after having had instruction in algebra and geometry. At this stage of advancement they are able to appreciate a scientific treatment of the subject and to make an intensive study of the same.

The sequence of topics is in accordance with the logical development of the subject. The first eight chapters may properly be considered *pure arithmetic,* since they deal primarily with the nature and laws of pure numbers. The remaining four chapters may be considered *applied arithmetic,* since they deal with numbers as applied to the business affairs of life. A pupil's skill in applied arithmetic depends in a large measure upon his knowledge of pure arithmetic.

Few rules have been given, but the fundamental principles have been prominently set forth, and model solutions, which serve to lead the pupil step by step through the process of reasoning and thereby compel him to think, are a special feature of the book. It is not the number of problems solved, but rather the number of principles mastered, that is worth while. Doubtless many a teacher has felt the truth of the following, taken from McLellan and Dewey's "Psychology of Number," p. 9: "It is a certain quality of practice, not mere practice, which produces the expert and the artist. Unless the practice is based upon rational principles, upon insight into facts and their meaning, 'experience' simply fixes incorrect acts into wrong habits. Non-scientific practice, even if it finally reaches sane and reasonable results—which is very unlikely—does so by unnecessarily long and circuitous routes; time and energy are wasted that might easily be saved by wise insight and direction at the outset." In the preparation of this text, the foregoing principles have been kept in mind.

The author gratefully acknowledges his indebtedness for helpful suggestions and criticisms to Mr. R. W. Fowler, Instructor in Arithmetic and Algebra in the Austin High School, and to Mr. A. N. McCallum, Superintendent of the Austin city schools.

B. F. SISK.

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CHAPTER I

BASIC PRINCIPLES: NOTATION AND NUMERATION

1. **Basic Principles.**—Every science rests upon certain basic principles, or primary truths, as a foundation. On taking up the study of a science, the learner should become familiar with the basic principles *at the beginning,* since their use is to guide him in reasoning from the known, or given, to the related unknown, or required. In mathematical science, the basic principles are in the form of *definitions, theorems* and *axioms.*

GENERAL DEFINITIONS

2. A **definition** (Latin *definire,* to bound or limit) is such a description of any thing as will distinguish it from all other things.

3. **Quantity** (Latin *quantus,* how much) is any thing which can be increased or diminished; it embraces number and magnitude.

4. **Science** (Latin *scire,* to know) is knowledge properly organized.

5. **Mathematics** is the science of quantity.

6. **Arithmetic** (Greek *arithmos,* number) is the science of numbers.

7. A **unit** (Latin *unus,* one) is one thing, or a group of things regarded as a single thing.

A *unit* is any standard of reference employed in counting any collection of objects, or in measuring any magnitude.—J. C. GLASHAN.

8. **Number** (Latin *numerare,* to count) is the ratio of one quantity to another quantity of the same kind taken as a unit.

"Number in the strict sense is the measure of quantity. It definitely measures a given quantity by denoting how many units of measurement make up the quantity." *

9. A **concrete number** (Latin *concrescere,* to grow together) is a number applied to some particular unit; as,

4 quarts, 7 dollars, etc.

NOTE.—In the strict sense, number is always abstract. It is a ratio, and answers the question, *how many?* In the examples above, "4" and "7" tell *how many,* and "quarts" and "dollars" tell *of what.* The idea of number is expressed by "4" and "7," not by "quarts" and "dollars."

When the unit is named, however, it is convenient and customary to speak of a number as concrete, to distinguish from *pure number,* which is always abstract.

10. An **abstract number** (Latin *abstrahere,* to draw away) is a number used without reference to any particular unit; as,

4, 7, 39, etc.

---

* *The Psychology of Number,* by McLellan and Dewey, p. 93.

11. A **principle** (Latin *principium,* a beginning, or origin) is a general truth.

12. A **problem** (Greek *problema,* a question) is a question offered for solution.

13. A **solution** (Latin *solvere,* to loosen) is a clear statement showing how the result is obtained.

14. An **axiom** (Greek *axioma,* a requisite) is a self-evident truth.

The following are the axioms most frequently used in mathematics:

1. The whole is greater than any of its parts.
2. The whole is equal to the sum of all its parts.
3. If equals are added to equals, the sums are equal.
4. If equals are subtracted from equals, the remainders are equal.
5. If equals are added to unequals, the sums are unequal.
6. If equals are subtracted from unequals, the remainders are unequal.
7. If equals are multiplied by the same number, the products are equal.
8. If equals are divided by the same number, the quotients are equal.
9. If things are equal to the same thing, they are equal to each other.
10. The same parts of equals are equal.
11. Equal powers of equals are equal.
12. Equal roots of equals are *numerically* equal.

15. Notation (Latin *notare,* to mark) is the art of representing numbers by means of symbols. It is intimately connected with numeration, which is generally defined as the art of reading numbers. The ability to read numbers is usually *implied* in the ability to write them. At the present time in this country, only two methods of notation are in general use. The first to be considered is the

ROMAN NOTATION

16. In the **Roman Notation,** seven capital letters are used. This method is rarely followed, except in numbering chapters and divisions of books, and on the dials of clocks and watches.

Letters:	I,	V,	X,	L,	C,	D,	M.
Values:	1,	5,	10,	50,	100,	500,	1000.

17. The following **principles** are followed in combining the letters:

1. Repeating a letter repeats its value; as,
   II = 2; XX = 20; XXX = 30.
2. When a letter is placed *before* another of greater value, its value is taken from that of the greater; as,
   IV = 4; IX = 9; XL = 40.
3. When a letter is placed *after* another of greater value, its value is added; as,
   VI = 6; XI = 11; LX = 60.
4. When a letter is placed *between* two letters of greater value, its value is *taken from the following letter,* not added to the preceding; as,
   XIV = 14, *not* 16.

5. A bar placed over a letter increases its value *a thousand times;* as,

$$\overline{\mathrm{V}}=5000;\quad \overline{\mathrm{L}}=50000;\quad \overline{\mathrm{M}}=1000000.$$

ARABIC NOTATION

18. The **Arabic Notation** is now more widely used than any other method. It is the simplest and clearest system of notation known to civilization. It received its name from the Arabs, who introduced the system into Christian Europe about the year 1200. The symbols, except the zero, originated in India before the Christian era; therefore, the term *Hindu* is sometimes used instead of *Arabic.* These symbols were not generally taught and used until the fifteenth century. In 1478 the first printed arithmetic appeared at Treviso, and in 1482 the first German arithmetic, at Bamberg, and these explained the system. In its present state of perfection, ten figures are employed in expressing numbers.

Figures:	**0,**	**1,**	**2,**	**3,**	**4,**	**5,**	**6,**	**7,**	**8,**	**9.**
Names:	**zero,**	**one,**	**two,**	**three,**	**four,**	**five,**	**six,**	**seven,**	**eight,**	**nine.**

The superiority of this system is due to the use of the zero, which renders possible the distinctive feature known as *local,* or *place value.* Each of the nine digits has two values—a **simple** value and a **local** value. The *simple* value of a figure is the value indicated by its *name.* A figure always has its simple value when standing alone or in units order. The *local* value of a figure is its value when placed in any order except units.

Thus, in 703, the simple value of "7" is *seven,* but the local value of "7" is *seven hundred.*

19. The **principles** involved in writing numbers are:

1. Ten units of any order make one of the next higher order.
2. Moving a significant figure one place to the left increases its value tenfold; one place to the right decreases its value tenfold.
3. Vacant orders in a number are filled with zeros.

20. Index notation may also be mentioned as a special form of the above system. Scientists often find it convenient in approximations to introduce powers of ten.

Thus, 246000000000 may be written 246×10^9, or 24.6×10^{10}.

Since $10^{-1} = 0.1$, and $10^{-2} = 0.01$, etc., a decimal fraction may be expressed by using 10 with a negative exponent.

E.g., 0.0000000375 may be written 37.5×10^{-9}.

These numbers may be multiplied thus,

$$246 \times 37.5 \times 10^{9-9} = 246 \times 37.5 = 9225.$$

COMMON OR FRENCH METHOD OF NUMERATION

21. The **Common** or **French Method of Numeration** is the method used by the people of the United States and of France. The figures are considered as far as possible in groups of *three* figures each, commencing at the right hand. The groups are called **periods.** The first twelve periods are as follows:

1. units
2. thousands
3. millions
4. billions
5. trillions
6. quadrillions
7. quintillions
8. sextillions
9. septillions
10. octillions
11. nonillions
12. decillions.

ENGLISH METHOD OF NUMERATION

22. The English people and people of many other European nations consider a period as made up of *six* figures. This method of grouping the figures is called the **English Method.** The first six periods according to this method are as follows:

1. units
2. millions
3. billions
4. trillions
5. quadrillions
6. quintillions

By the English Method, the number 234179386564 is read *two hundred thirty-four thousand one hundred seventy-nine million, three hundred eighty-six thousand, five hundred sixty-four.* By the French Method, it would read *two hundred thirty-four billion, one hundred seventy-nine million, three hundred eighty-six thousand, five hundred sixty-four.*

SCALES OF NOTATION

23. The **scale** (Latin *scala,* a ladder), in any system of notation, is the succession of ascending and descending order values.

24. The **radix** of a scale (Latin *radix,* root) is the number of units which it takes of one order to make one unit of the next higher order. The radix is also called the *base-number.* In the Arabic Notation the radix is ten and is uniform. In any scale the number of characters, including 0, is the same as the number of units in the radix of that scale. The scale gets its name from the number of units in its radix. To express numbers in scales higher than the decimal, new characters must be employed. Thus, *a* may be used to represent ten, *b*, eleven, *c*, twelve, etc.

Scale.	*Characters Used.*	*Radix.*		
Duodecimal	1, 2, 3, 4, 5, 6, 7, 8, 9, *a*, *b*, 0	twelve,	expressed	10
Undenary	1, 2, 3, 4, 5, 6, 7, 8, 9, *a*, 0	eleven,	"	10
Decimal	1, 2, 3, 4, 5, 6, 7, 8, 9, 0	ten,	"	10
Nonary	1, 2, 3, 4, 5, 6, 7, 8, 0	nine,	"	10
Octary	1, 2, 3, 4, 5, 6, 7, 0	eight,	"	10
Septenary	1, 2, 3, 4, 5, 6, 0	seven,	"	10
Senary	1, 2, 3, 4, 5, 0	six,	"	10
Quinary	1, 2, 3, 4, 0	five,	"	10
Quaternary	1, 2, 3, 0	four,	"	10
Ternary	1, 2, 0	three,	"	10
Binary	1, 0	two,	"	10

25. Inasmuch as the names of the *orders* used in expressing numbers are adapted to the decimal scale, numbers expressed in other scales should be read by naming the number of units in each order. For example, the number 256 in the *octary scale* should be read: *octary scale,* 2 *units of the third order,* 5 *of the second, and* 6 *of the first.* Usually a small subscript is used to indicate the radix. Thus, 423_5 means 423 in the quinary scale.

EXAMPLES

26. Write in the senary scale the numbers corresponding to the numbers from 1 to 15 in the decimal scale.

Explanation.—The radix is 6; therefore, the figures used are 1, 2, 3, 4, 5, 0. It must be remembered that *when the number of units in any order reaches* SIX, *it makes one of the next higher order.*

Radix ten : 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15.
Radix six : 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23.

In like manner, write in other scales the numbers corresponding to the numbers from 1 to 35 in the decimal scale.

27. To change from any scale to the decimal scale:

Example.—Express 3432_6 in the decimal scale.

SOLUTION:

$$3 \times 6^3 + 4 \times 6^2 + 3 \times 6^1 + 2 = 648 + 144 + 18 + 2 = 812.$$
$$\therefore\ 3432_6 = 812_{10}.$$

This example may be treated as one in compound numbers. Thus,

OPERATION.

$$\begin{array}{r} 3432 \\ 6 \\ \hline 22 \\ 6 \\ \hline 135 \\ 6 \\ \hline 812 \end{array}$$

EXPLANATION.—Since each higher unit is equal to six of the next lower order, 3 units of the fourth order are equal to 18 of the third. By adding 4, the number of the third order given, we obtain 22, the number of the third order. Proceeding in the same manner, until the number of units of the first order is obtained, the number in the decimal scale is 812.

28. To change from the decimal to another scale:

Example.—Change 5287 from the decimal to the quinary scale.

OPERATION.

$$\begin{array}{r|l} 5 & 5287 \\ \hline 5 & 1057\text{, rem. } 2 \\ \hline 5 & 211\text{, rem. } 2 \\ \hline 5 & 42\text{, rem. } 1 \\ \hline 5 & 8\text{, rem. } 2 \\ \hline & 1\text{, rem. } 3 \end{array}$$

EXPLANATION.—By dividing by 5, we obtain the number of units of the second order and the number of units of the first order remaining. By continuing to divide by 5, the number of units in the successive orders is obtained, and the number of units remaining after division. It is thus found that $5287_{10} = 132122_5$.

29. Example.—Change $.324_5$ to the decimal scale.

SOLUTION:

$$.324_5 = \tfrac{3}{5} + \tfrac{2}{5^2} + \tfrac{4}{5^3} = \text{some fraction.}$$

Let f denote the fraction.

$$\begin{aligned}
\text{Then } f &= \tfrac{3}{5} + \tfrac{2}{5^2} + \tfrac{4}{5^3}.\\
5f &= 3 + \tfrac{2}{5} + \tfrac{4}{5^2}.\\
25f &= 15 + 2 + \tfrac{4}{5}.\\
125f &= 75 + 10 + 4 = 89.\\
f &= \tfrac{89}{125} = .712, \text{ the required fraction.}
\end{aligned}$$

$$\therefore\ .324_5 = .712_{10}.$$

30. Example.—Change $.712_{10}$ to a decimal in the scale of 5.

OPERATION:

$$\begin{array}{rl}
.712 & \\
5 & \\
\hline
3.560 & \text{5ths.}\\
5 & \\
\hline
2.80 & \text{25ths.}\\
5 & \\
\hline
4.0 & \text{125ths.}
\end{array}$$

$$\therefore\ .712_{10} = \tfrac{3}{5} + \tfrac{2}{5^2} + \tfrac{4}{5^3} = .324_5.$$

31. The operations, *addition, subtraction, multiplication,* and *division,* may be performed with numbers in any scale. The processes are the same as in the decimal scale. The student must bear in mind every time the number of units in each order required to make one unit of the next higher order.

(1)

Add 245_6 and 432_6.

OPERATION.

$$\begin{array}{r}
245_6\\
432_6\\
\hline
1121_6
\end{array}$$

(2)

Subtract 243_5 from 432_5.

OPERATION.

$$\begin{array}{r}
432_5\\
243_5\\
\hline
134_5
\end{array}$$

(3)

Multiply 312_7 by 234_7.

OPERATION.

$$\begin{array}{r}
312_7 \\
\underline{234_7} \\
1551 \\
1236 \\
\underline{624} \\
106641_7
\end{array}$$

(4)

Divide 106641_7 by 312_7.

OPERATION.

$$\begin{array}{l}
312_7)106641_7(234_7 \\
\underline{624} \\
1424 \\
\underline{1236} \\
1551 \\
\underline{1551}
\end{array}$$

COMPARISON OF SCALES

The first fifteen numbers in the following scales are:

Duodecimal,	1,	2,	3,	4,	5,	6,	7,	8,	9,	a,	b,	10,	11,	12,	13.
Undenary,	1,	2,	3,	4,	5,	6,	7,	8,	9,	a,	10,	11,	12,	13,	14.
Decimal,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10,	11,	12,	13,	14,	15.
Nonary,	1,	2,	3,	4,	5,	6,	7,	8,	10,	11,	12,	13,	14,	15,	16.
Octary,	1,	2,	3,	4,	5,	6,	7,	10,	11,	12,	13,	14,	15,	16,	17.
Septenary	1,	2,	3,	4,	5,	6,	10,	11,	12,	13,	14,	15,	16,	20,	21.
Senary	1,	2,	3,	4,	5,	10,	11,	12,	13,	14,	15,	20,	21,	22,	23.
Quinary	1,	2,	3,	4,	10,	11,	12,	13,	14,	20,	21,	22,	23,	24,	30.
Quaternary,	1,	2,	3,	10,	11,	12,	13,	20,	21,	22,	23,	30,	31,	32,	33.
Ternary,	1,	2,	10,	11,	12,	20,	21,	22,	etc.						
Binary,	1,	10,	11,	100,	101,	110,	111,	etc.							

Exercise I

1. Define arithmetic, number, unit, principle, and axiom. What purpose do the axioms serve? In what year was the first arithmetic printed?

2. What is notation? What two systems of notation are now in common use? State the five principles of the Roman notation; the three of the Arabic notation.

3. Express 23479, 5087, and 355 by the Roman notation.

4. Express XXIX, XCIX, $\overline{\text{CXIV}}$ by the Arabic notation.

5. Express 24000 and 0.0000017 by the *index notation.* To what kind of numbers is the index notation adapted?

6. Read by the French method 342378921476; read the same number by the English method.

7. Define scale and radix. From what does the scale get its name?

8. Write the first sixty numbers in each scale.

9. Change the following to the decimal scale:

(1) 5867_9; (2) 23123_4; (3) $231a5_{11}$.

10. Change 58375 from the decimal (1) to the octary scale, (2) to the ternary, (3) to the duodecimal, (4) to the senary.

11. Add 3123_5, 4124_5, 3243_5, 4233_5.

12. Subtract 34562_7 from 62456_7.

13. Multiply 3424_5 by 234_5.

14. Divide 2034431_5 by 3424_5.

CHAPTER II

THE FOUR FUNDAMENTAL OPERATIONS

I. ADDITION

32. Addition (Latin *addere,* to increase) is the process of finding the sum of two or more numbers. It is a short method of *counting.*

33. The numbers to be added are called **addends.**

34. The result of addition is called the **sum** or **amount.**

35. The **sign of addition** (+) is read *plus,* meaning *more.*

36. The **sign of equality** (=) is read *equals.*

NOTE.—The signs (+) and (−) were used by Johannes Widman in his *Mercantile Arithmetic*, published in Leipzig in 1489. He used them, however, to denote *excess* and *deficiency*. These symbols were first used to denote *addition* and *subtraction* by a German mathematician (Michael Stifel) in his work published in 1544.

The sign of equality (=) was first used by the English mathematician, Robert Recorde, in an algebra called the *Whetstone of Witte*, published in 1557.

37. Fundamental principles used in addition:

1. Only similar addends can be added; therefore, the addends must have the *same unit.*
2. The sum is similar to the addends, and contains all the units of all the addends.
3. The sum is the same regardless of the order of grouping the addends.

4. The sum is the same regardless of the order of performing the operation.

NOTE.—Principle 3 is called the **associative law of addition**; principle 4, the **commutative law of addition.**

38. To test the accuracy of addition:

1. Review the work.
2. Add the columns in reverse order.
3. Cast out the 9's.

39. RULE: **To cast the 9's out of any number:**

Divide the sum of the digits by 9, *and find the excess.*

Thus, to find the excess of 9's in 23784, begin at the left: $2+3+7=12$; cast out the 9 and say $3+8=11$; cast out the 9 and say $2+4=6$; the excess is 6. Or, $2+3+7+8+4=24$; $24 \div 9=2$, with a remainder 6.

The test of accuracy by casting out the 9's is based upon the principle that *the excess of 9's in any number equals the excess of 9's in the sum of its digits.*

This may be shown by the following

ILLUSTRATION:

$$4352 = \left\{\begin{array}{rrrr} 4000=4\times & 1000=4\times & (999+1)=4\times & 999+4 \\ 300=3\times & 100=3\times & (99+1)=3\times & 99+3 \\ 50=5\times & 10=5\times & (9+1)=5\times & 9+5 \\ 2=2\times & 1=2\times & (0+1)=2\times & 0+2 \end{array}\right.$$

$\therefore\ 4352 =$ the above multiples of $9 + 14$

Observe that the number has been separated into multiples of 9, and the sum of the digits of the number. There can be no excess when the multiples are divided by 9, so *the only excess must be in the sum of the digits.*

It must be remembered, however, that this test fails to discover any error not affecting the excesses of 9's, such as an interchange of digits, the addition or the subtraction of 9, etc., but such errors are rare.

EXAMPLE

40. Add and test: 3476+2562+1653+8721.

Excess in 3476.................. 2
" " 2562.................. 6
" " 1653.................. 6
" " 8721.................. 0
" " $\overline{16412}$ = 5 = excess in....$\overline{14}$ = 5.

Study the example above, and write out the rule for testing the accuracy of addition by casting out the 9's.

Exercise II

Add and test by casting out the 9's.

(1)

States.	*Population.*
Alabama	1,828,697
Arizona T.	122,931
Arkansas	1,311,564
California	1,485,053
Colorado	539,700
Connecticut	908,420
Delaware	184,735
Dist. of Columbia	278,718
Florida	528,542
Georgia	2,216,331
Idaho	161,772
Illinois	4,821,550
Indiana	2,516,462

(2)

States.	*Population.*
Iowa	2,231,853
Kansas	1,470,495
Kentucky	2,147,174
Louisiana	1,381,625
Maine	694,466
Maryland	1,188,044
Massachusetts	2,805,346
Michigan	2,420,982
Minnesota	1,751,394
Mississippi	1,551,270
Missouri	3,106,665
Montana	243,329
Nebraska	1,066,300

(3)

States.	*Population.*
Nevada	42,335
New Hampshire	411,588
New Jersey	1,883,669
New Mexico T.	195,310
New York	7,268,894
North Carolina	1,893,810
North Dakota	319,146
Ohio	4,157,545
Oklahoma T.	398,331
Oregon	413,536
Pennsylvania	6,302,115
Rhode Island	428,556
South Carolina	1,340,316

(4)

States.	*Population.*
South Dakota	401,570
Tennessee	2,020,616
Texas	3,048,710
Utah	276,749
Vermont	343,641
Virginia	1,854,184
Washington	518,103
West Virginia	958,800
Wisconsin	2,069,042
Wyoming	92,531
Alaska T.	63,592
Hawaii T.	154,001
Indian T.	392,060

Note.—The foregoing statistics are taken from the census of 1900.

II. SUBTRACTION

41. **Subtraction** (Latin *subtrahere,* to take away) is the process of finding the difference between two numbers, or of taking one number from another of the same kind.

42. The **minuend** (Latin *minuere,* to lessen) is the number from which another number is taken.

43. The **subtrahend** is the number to be taken from the minuend.

44. The **remainder** (Latin *remanere,* to stay back) is the result obtained by taking the subtrahend from the minuend.

45. The **sign of subtraction** (−) is read *minus,* meaning *less.*

Note.—See note under 36.

46. Fundamental principles used in subtraction:

1. The minuend, subtrahend, and remainder must be similar numbers.
2. The remainder equals the minuend minus the subtrahend.
3. The subtrahend equals the minuend minus the remainder.
4. The minuend equals the sum of the subtrahend and remainder.

NOTE.—Subtraction is the **inverse** of addition; therefore, the **associative** and **commutative laws** hold in subtraction. (See 37.)

47. To test the accuracy of subtraction:

1. Review the work.
2. Add the subtrahend and remainder; the sum should equal the minuend.
3. Cast out the 9's.

Since subtraction is the inverse of addition, the second test is the simplest. One example, however, will be given to show how to test the work by casting out the 9's.

EXAMPLE

48. Subtract and test: 76212168 − 62622250.

$$\begin{array}{lrl} \text{Excess in} & 76212168 & = 6 \\ \text{`` \quad ``} & \underline{62622250} & = 7 \\ \text{`` \quad ``} & 13589918 & = 8 \\ \therefore \text{ Excess in} & (8 + 7) & = \text{excess in } 6. \end{array}$$

Study the example above and *write the rule.*

Exercise III

Subtract and test:

1. From 71086540 take 64179730.

2. From 11437687 take 7834539.

3. Which is the nearer number to 920736; 1816045 or 25427?

4. The difference of two numbers is 9034, and the greater is 75421; what is the less?

5. From 64782 take 35846, beginning at the left instead of at the right.

III. MULTIPLICATION

49. Multiplication (Latin *multiplicare,* to increase in number) is the process of finding the product of two numbers. It is a short method of addition where the addends are equal. Multiplication may also be defined as a short method of *counting.*

EXAMPLE.—In a room there are five rows of desks, and in each row are eight desks. How many desks in the room ?

1. *Count* the desks ; 40 desks are in the room.

2. *Add:* 8 desks + 8 desks + 8 desks + 8 desks + 8 desks = 40 desks.

3. *Multiply:* 1 row contains 8 desks, 5 rows contain 5 × 8 desks = 40 desks.

Each of the three operations produces the same result.

50. The **multiplicand** (Latin *multiplicandus,* to be multiplied) is the number to be multiplied; it is the number to be used as the *addend.*

51. The **multiplier** is the number that shows how many times the multiplicand is to be taken.

52. The **product** (Latin *productus,* produced) is the result obtained from multiplying one number by another.

53. The **factors of the product** (Latin *facere,* to make) are the multiplicand and the multiplier.

54. The **sign of multiplication** (×) is read *times* or *multiplied by.*

1. It is read *times,* when the multiplier precedes it.
2. It is read *multiplied by,* when the multiplier follows it.

Note.—This sign (×) was introduced by an English mathematician, William Oughtred, in his arithmetic called *Clavis Mathematica.* This work was published in 1631.

55. Fundamental principles used in multiplication:

1. The multiplier is always a pure number (abstract).
2. The product is always the same in kind as the multiplicand. (This is due to principle 2, **37.**)
3. The product is the same regardless of the order of grouping the factors.
4. The product is the same whichever factor is made the multiplier.

Note.—Principle 3 is called the **associative law of multiplication;** principle 4, the **commutative law of multiplication.**

56. To test the accuracy of multiplication:

1. Review the work.
2. Divide the product by either factor; the quotient should be the other.

3. Cast out the 9's.

One example will be given as an illustration of test 3.

57. Multiply and test: 3765×67.

$$\left.\begin{array}{lr} \text{Excess in} & 3765 = 3 \\ \text{“\quad “} & 67 = 4 \end{array}\right\} 3 \times 4 = 12;\ \text{excess, } 3.$$

$$\begin{array}{lr} & \overline{26355} \\ & \underline{22590} \\ \text{Excess in} & 252255 = 3. \end{array}$$

Study the example above and *write the rule.*

SHORT PROCESSES IN MULTIPLICATION

58. To multiply by 10, 100, 1000, etc.

According to principle 2, **19,** we may simply annex as many zeros to the multiplicand as there are in the multiplier.

Thus, 642 multiplied by 10 = 6420.
84.6 " " 100 = 8460.

59. To multiply by an aliquot part of 10, 100, etc.

To multiply by $12\frac{1}{2}$, $16\frac{2}{3}$, 25, $33\frac{1}{3}$, is the same as to multiply by $\frac{100}{8}$, $\frac{100}{6}$, $\frac{100}{4}$, $\frac{100}{3}$. It is easier to multiply by $\frac{100}{8}$, that is, to add two zeros and divide by 8, than to multiply by $12\frac{1}{2}$, and similarly for the other cases. These short processes are especially valuable, because the above numbers are common rates in business.

Example.—$4293 \times 33\frac{1}{3} = ?$

Solution : $33\frac{1}{3} = \frac{100}{3} = \frac{1}{3}$ of 100.
$\therefore\ 33\frac{1}{3} \times 4293 = \frac{1}{3}$ of $429300 = 143100$.

Various combinations may be made. Thus,

1. Multiply 426 by $1.16\frac{2}{3}$.

Solution : $1.16\frac{2}{3} \times 426 = 426 + \frac{1}{6}$ of $426 = 497$.

2. Multiply 63 by $15\frac{2}{7}$.

Solution:

$$\begin{array}{rr} 14\frac{2}{7} \times 63 = \frac{1}{7} \text{ of } 6300 = & 900. \\ 1 \times 63 = & 63. \\ \hline \therefore\ 15\frac{2}{7} \times 63 = & 963. \end{array}$$

The student should make other combinations.

60. To multiply when one part of the multiplier is a multiple of another part.

Example.—Multiply 643 by 428.

Solution:

$$\begin{array}{rl} 643 & = \text{the multiplicand.} \\ 428 & = \text{the multiplier, with } 28 = 4 \times 7. \\ \hline 2572 & = \text{the product by 4 (}\textit{hundreds}\text{).} \\ 18004 & = 7 \times 4 \text{ times the multiplicand.} \\ \hline 275204 & = \text{the product by 428.} \end{array}$$

61. To multiply when the multiplier differs but little from some power of ten.

To multiply by 998 is to multiply by (10^3-2); that is, to annex 3 zeros and to subtract 2 times the multiplicand.

Example.—Multiply 3642 by 998.

Solution:

$$\begin{array}{rl} 3642 & = \text{the multiplicand.} \\ 998 & = \text{the multiplier.} \\ \hline 3642000 & = \text{the product by } 10^3. \\ 7284 & = \text{the product by 2.} \\ \hline 3634716 & = \text{the product by } 10^3 - 2\text{, or 998.} \end{array}$$

62. To square a number ending in 5.

To multiply 85 by 85, we may merely say,

$$8 \times 9 = 72,\quad 5 \times 5 = 25,\quad \therefore\ 85 \times 85 = 7225.$$

To prove that this process is general:

1. Any number ending in 5 may be represented by $10x+5$, where x may equal any number.

2. $(10x+5)^2 = 100x^2 + 100x + 25.$

$$= 100x(x+1) + 25.$$

3. That is, the result ends in 25, and the number of hundreds is $x(x+1)$.

Study the work above and *write the rule.*

63. Approximations in multiplication. In business calculations, results beyond two or three decimal places are seldom desired, and in science absolutely accurate measurements are rarely possible; therefore, in many cases a result which is correct to the 2d or 3d decimal place is sufficiently accurate.

Example.—The diameter of a circle is 12.37 feet; find the circumference, correct to the 2d decimal place.

Solution:

12.37 ft. = the multiplicand.
3.1416 = the multiplier = the ratio of circum. to diameter.
37.11 = 3 × the multiplicand.
1.237 = 0.1 × " "
.494 = 0.04 × " "
.012 = 0.001 × " "
.007 = 0.0006 × " "
38.86 = 3.1416 × " "

Note.—In order to be sure that the result is correct to the 2d decimal place, some of the partial products are carried to the 3d decimal place, and the effect of the 4th decimal place upon the 3d is kept in mind.

By the use of this method, much labor is saved in the solution of problems involving large decimals, as those in compound interest. A little practice will make the work very easy for the student.

Exercise IV

Multiply and test by casting out the 9's:

1. 34659 by 5346.
2. 1283.74 by 3.1416.

3. 86497 by 980009.
4. 56789 by 12345.

Using the shortest process, multiply:

5. 476 by 1000; 3700 by 1000.
6. 4324 by $12\frac{1}{2}$; 48636 by $16\frac{2}{3}$.
7. 499 by 25; 37812 by $33\frac{1}{3}$.
8. 3884 by 125; 1284 by $133\frac{1}{3}$.
9. 3996 by $15\frac{2}{3}$; 6993 by $134\frac{1}{3}$.
10. 3782 by 248; 1817 by 642.
11. 75 by 75; 95 by 95.
12. 1292 by 995; 3876 by 97.
13. Prove that to multiply a number, say 4842, by 625, you may annex 4 zeros and divide by 2 four times.
14. Multiply 27.41623 by 1.42352, retaining only 3 decimal places in the product.
15. The compound interest on $1 at $4\frac{1}{2}\%$ for 3 years is $0.14116612; find the compound interest on $378.25 at the same rate and for the same time, correct to 1 cent.
16. The diameter of a circular lot is 10.1321 feet; find the distance around it, correct to one-tenth of a foot.

IV. DIVISION

64. Division (Latin *dividere,* to divide) is the process of finding the quotient of two numbers.

Division is the **inverse** of multiplication; that is, it is the process of finding one factor when the product and the other factor are given.

NOTE.—When the quotient sought is an abstract number, the division is called **subtractional division.** The quotient, then, is the number of times that the divisor can be subtracted from the dividend. This process shows that division is a short method of subtraction.

When the quotient sought is a concrete number, the division is called **partitive division.** The divisor, then, is the number of parts into which the dividend is to be divided. The quotient represents the size of the parts.

65. The **dividend** (Latin *dividendum,* the thing to be divided) is the number to be divided.

66. The **divisor** is the number to divide by. It is the known factor.

67. The **quotient** (Latin *quoties,* how many times) is the result of the division. It is the factor sought.

68. The **sign of division** (÷) is read *divided by.*

NOTE.—This sign (÷) as a symbol of division was first used by Rahn in an algebra published at Zurich in 1659.

69. Fundamental principles used in division:

1. When the dividend and divisor are like numbers. the quotient is an abstract number.
2. When the divisor is an abstract number, the quotient is like the dividend.
3. The dividend is equal to the product of the quotient and the divisor.
4. If the dividend and the divisor be multiplied by

the same number, or divided by the same number, the quotient remains unchanged.

NOTE.—Division is the **inverse** of multiplication; therefore, the **associative** and **commutative laws** hold in division. (See 55.)

70. To test the accuracy of division:

1. Review the work.
2. Apply principle 3.
3. Cast out the 9's.

One example will be given as an illustration of test 3.

EXAMPLE

71. Divide and test: 5864 ÷ 86.

SOLUTION:

5864 ÷ 86 = 68, with remainder 16.

Now, 86 × 68 + 16 = 5864.

$$\left.\begin{array}{lr} \text{Excess in} & 86=5 \\ \text{“ \quad “} & 68=5 \\ \text{“ \quad “} & 16=7 \\ \text{“ \quad “} & 5864=5 \end{array}\right\} 5\times5+7=32;\ \text{excess, } 5 = \text{excess in } 5864.$$

Study the example above and *write the rule.*

SHORT PROCESSES IN DIVISION

72. RULE.—**To divide by some power of 10:**

Cut off as many figures in the dividend as there are zeros in the divisor.

The figures cut off will be the remainder, and the others the quotient.

Example.—Divide 34793 by 100.

SOLUTION:—34793 ÷ 100 = 347, with remainder 93.

73. To divide by a composite number.

Example.—Divide 1728 by 36.

Since $36=3\times3\times2\times2$, we may divide by the factors; thus,

OPERATION.

$$\begin{array}{r}
2)\underline{1728} \\
2)\underline{864} \\
3)\underline{432} \\
3)\underline{144} \\
48
\end{array}
\qquad \therefore 1728 \div 36 = 48.$$

74. To divide by an aliquot part of 10, 100, 1000, etc.

To divide by $2\frac{1}{2}$, $12\frac{1}{2}$, $16\frac{2}{3}$, 25, $33\frac{1}{3}$, etc., is the same as to divide by $\frac{10}{4}$, $\frac{100}{8}$, $\frac{100}{6}$, $\frac{100}{4}$, $\frac{100}{3}$, etc.

RULE.—**To divide by an aliquot part of 10, 100, 1000, etc.:**

Multiply by the denominator and cut off as many figures from the right as there are zeros in the numerator.

Example.—Divide 58200 by $12\frac{1}{2}$.

SOLUTION.—$12\frac{1}{2} = \frac{100}{8}$; $58200 \div \frac{100}{8} = \frac{1}{100}$ of $8 \times 58200 = 4656$.

75. Approximations in Division.—In a problem involving large decimals, an approximate answer is usually all that is desired, and in some cases all that is possible. Much time and labor may be saved by using the following method.

Example.—Divide 2.614746 by 1.123456, correct to the second decimal place.

OPERATION.

$$\begin{array}{r l}
1.123\cancel{4}\cancel{5}\cancel{6})2.614\cancel{7}\cancel{4}\cancel{6}(2.32 & \\
\underline{2.247} & \text{(1 carried)} \\
367 & \\
\underline{337} & \text{(1 carried)} \\
30 & \\
\underline{22} &
\end{array}$$

EXPLANATION.—Since only three figures are required in the answer, all but four may be neglected in each, the dividend and divisor. The figures *marked* will not affect the answer required, if proper allowance be made for the effect they may have upon the partial product. Observe that 1 is added to the 6, making 7 in

the first partial product. After each division one figure of the divisor is neglected, but its effect upon the partial product is kept in mind.

A TEST FOR THE FOUR FUNDAMENTAL OPERATIONS.

76. The following is a very simple test of the accuracy of *addition*, *subtraction*, *multiplication*, and *division*. In fact, it is a modification of the method of casting out the 9's. It may be called the **unitate method**. The *unitate* of a number is the sum of its digits reduced to units order. Note these examples:

The unitate of 32798 = 29 = 11 = 2.
" " " 47931 = 24 = 6.
" " " 28645 = 25 = 7.

The sum of the digits of the first number is 29; these digits added = 11, and these added = 2. The unitate of 32798 = 2. Note the following example in multiplication:

The unitate in 242 = 8 } 6 × 8 = 48 = 12 = 3.
" " " 132 = 6 }
484
726
242
The unitate in 31944 = 21 = 3.

EXPLANATION.—If the work is correct, the unitate of the product equals the unitate of the product of the unitates of the multiplicand and multiplier. It is not necessary to write down as many figures as are written above. The unitate of each number can easily be found mentally.

Try this method with the following:

1. Add 37864, 33977, 47693, 36821.
2. Subtract 34689 from 78601.
3. Multiply 46031 by 527; 59993 by 587.
4. Divide 5332 by 124; 4326422 by 961.

PRECEDENCE OF SIGNS

77. A number with no sign preceding it is considered positive. The signs take effect in the following order:

1. The signs of addition and subtraction take effect in the order in which they occur. Thus, $2 + 3 - 4 + 7 - 2 = 6$.
2. The signs of multiplication and division take effect in the order in which they occur. Thus, $24 \div 6 \times 2 = 8$, not 2.

NOTE.—Authorities differ on this point.

3. The signs of multiplication and division both take effect before the signs of addition and subtraction. Thus,

$$36 + 2 \times 3 - 8 \div 4 \times 2 = 36 + 6 - 4 = 38.$$

4. The above statements must be modified when braces, brackets, parentheses, or vinculums are used. The expressions in the braces, brackets, parentheses, and vinculums must be simplified first. Thus,

1. $24 \times (9 + 6) \div (\overline{28 - 18} \times 2) = 24 \times 15 \div 20 = 18.$

2. $$\begin{aligned} 6 \times \left\{4 - [3 + (5 - \overline{3 + 2})]\right\} &= 6 \times \left\{4 - [3 + (5 - 5)]\right\}. \\ &= 6 \times \left\{4 - [3 + 0]\right\}. \\ &= 6 \times \left\{4 - 3\right\}. \\ &= 6 \times 1 = 6. \end{aligned}$$

NOTE.—The distributive law for multiplication and division may be illustrated by the following: $4\,(6 - 2) = 4 \times 6 - 4 \times 2$, and $\frac{6 - 4}{2} = \frac{6}{2} - \frac{4}{2}$.

Exercise V

Divide and test by casting out the 9's:

1. 1554768 by 216; 1674918 by 189.
2. 65980064 by 5004; 47863 by 7525.

Find the quotient, correct to the second decimal place:

3. 58.140625 ÷ 7.625; 100 ÷ 3.14159.
4. 997.21567897 ÷ 37.7565936.

Find the quotient, correct to the nearest 1000:

5. 225000000 ÷ 46.55; 93000000 ÷ 245.
6. In a certain year the revenue of the United States Government was $403080983, which was $6.577 to each person; what was the population in that year?

Simplify:

7. $52 + 39 \div 3 - 42 \times 3 + 6$.
8. $(105 \div 21) + (80 \div 5 \times 81) + (36 \div 9)$.
9. $36 - [30 \div (3 + \overline{5 + 7}) + 24] + 10 \div 5$.
10. $96 \div 12 \times 2 + 15 \div 5 - 4 \times 2$.
11. $16 \times 4 \div 8 - 7 + 48 \div 16 - 3 - 7 \times 4 \times 0 \times 9 \times 16 + 24 \times 6 \div 48 - 4 \times 9 \div 12$.

CHAPTER III

FACTORS AND MULTIPLES

DEFINITIONS

78. An **integer** (Latin *integer,* whole) is a whole number; as, 1, 2, 4, etc.

NOTE.—Only integral numbers are considered here. Integral numbers are either **prime** or **composite.**

79. A **prime number** (Latin *primus,* first) is a number that cannot be divided by any integer (except itself and 1) without a remainder; as 1, 2, 3, 5, 7, etc.

80. A **composite number** (Latin *componere,* to put together) is a number that can be exactly divided by some other integer than itself and 1; as, 4, 6, 9, etc.

Integral numbers are either **even** or **odd.**

81. An **even number** (Old English *efen,* level) is a number exactly divisible by 2. Even numbers always end in 0, 2, 4, 6, or 8.

82. An **odd number** (Old English *odde,* without a mate) is a number which, when divided by 2, has 1 for a remainder. Odd numbers always end in 1, 3, 5, 7, or 9.

83. A **factor** of a number (Latin *factor,* a maker) is any exact divisor of the number.

The factors of a number, when multiplied together, *make* the number.

NOTE.—In giving the factors of a number, it is customary to omit 1 and the *number itself.*

84. A **prime factor** is a prime number that is a factor.

85. A **common factor** of two or more numbers is a number that will exactly divide each of them.

86. The **highest common factor** (h. c. f.) of two or more numbers is the highest number that will exactly divide each of them.

NOTE.—The terms, *divisor*, *factor*, *measure*, and *submultiple*, are commonly used to mean the same thing.

87. A **multiple** of a number (Latin *multus*, many, and *plus*, more) is any integral number of times the number. Thus, 16 is a multiple of 4, because it is a *number of times* 4.

88. A **common multiple** of two or more numbers is a number that will exactly contain each of them.

89. The **lowest common multiple** (l. c. m.) of two or more numbers is the lowest number that will exactly contain each of them.

TESTS OF DIVISIBILITY

90. Illustrate each of the following statements:

1. 2 is a factor of any even number.
2. 3 is a factor of any number the sum of whose digits is divisible by 3.
3. 4 is a factor of a number if the number expressed by its two right hand figures is divisible by 4.
4. 5 is a factor of any number whose right hand figure is 0 or 5.
5. 6 is a factor of any *even* number the sum of whose digits is divisible by 3.

6. 8 is a factor of a number if the number expressed by its three right hand figures is divisible by 8.
7. 9 is a factor of any number the sum of whose digits is divisible by 9.
8. 10 is a factor of any number whose right hand figure is 0.
9. 11 is a factor of a number if the sum of the digits in the *odd places* minus the sum of the digits in the *even places* is 0 or a multiple of 11. 24629 is divisible by 11, for (9+6+2) – (2+4)=11.

I. HIGHEST COMMON FACTOR

91. The **highest common factor** of two or more numbers is commonly called the greatest common divisor (g. c. d.).

92. Fundamental principles used in finding the h. c. f.:

1. A factor of a number is a factor of any multiple of that number.
2. A factor of each of two numbers is a factor of their sum.
3. A factor of each of two numbers is a factor of their difference.

93. The **method of factoring** may be illustrated by the following

Example: Find the h. c. f. of 24, 36, 48, and 60.

Solution:

$$\text{I.}\left\{\begin{array}{ll} 1. & 24 = 2 \times 2 \times 2 \times 3. \\ 2. & 36 = 2 \times 2 \times 3 \times 3. \\ 3. & 48 = 2 \times 2 \times 2 \times 2 \times 3. \\ 4. & 60 = 2 \times 2 \times 3 \times 5. \end{array}\right.$$

II.

1. Since 2 is found *twice* in each, the h. c. f. must contain *two* 2's.
2. Since 3 is found *once* in each, the h. c. f. must contain *one* 3.
3. Since no other factors are *common*, the h. c. f. must contain *no other factors*.
4. Therefore, the h. c. f. of 24, 36, 48, and 60 = 12.

With small numbers, this method is preferable. With large numbers, the long division method is preferable. This method is sometimes called the **Euclidean method,** from Euclid, who used it about 300 B.C.

94. The **Euclidean** or **long division method** may be illustrated by the following

Example: Find the h. c. f. of 216 and 324.

OPERATION.

```
216)324(1
    216
    ---
    108)216(2
        216
        ---
```

EXPLANATION.—If 216 were the h. c. f., it would be necessary for it to be contained in 324 at least *twice*, and in their difference at least *once*. Since this is not the case, 216 is not the h. c. f. According to principle 3, 92, the h. c. f. cannot be greater than 108; therefore, we take 108 and 216. If 108 is the h. c. f., it must also be a factor of 216, which it is. 108 is therefore the h. c. f. of 216 and 324.

Exercise VI

By factoring find the h. c. f. of:

1. 15, 30, and 45
2. 116, 232, and 58.
3. 47, 141, and 705.
4. 20, 36, and 48.
5. 37, 111, and 74.
6. 17, 85, and 170.

Using long division, find the h. c. f. of:

7. 160, 256, and 1600.
8. 244, 1220, and 732.

9. 59, 295, and 177.
10. 765, 855, and 1035.
11. 175, 250, and 625.
12. 187, 921, and 888.

II. LEAST COMMON MULTIPLE

95. Fundamental principles to be observed:

1. The l. c. m. of two or more numbers cannot be less than the largest of the numbers; that is, it must contain all the prime factors of the largest of the numbers.
2. It must contain all the prime factors of the smaller numbers which are not factors of the largest.
3. It must contain no other factors.

96. The **method of factoring** may be illustrated by the following

Example: Find the l. c. m. of 36, 72, and 108.

OPERATION.

1. $36 = 2 \times 2 \times 3 \times 3$.
2. $72 = 2 \times 2 \times 2 \times 3 \times 3$.
3. $108 = 2 \times 2 \times 3 \times 3 \times 3$.

According to the first principle above, we must use all the factors of 108. Since the factor 2 is found in 72 one time more than in 108, we must use this factor again. But according to principle 3, no other factors must be used. Therefore, the l. c. m. is $2 \times 2 \times 2 \times 3 \times 3 \times 3 = 216$.

97. The **highest common factor** may be used in finding the least common multiple. Using the example above:

1. The h. c. f. of 36, 72, and 108 is 36.
2. This factor is used once and no more in the l. c. m.

3. Therefore, the factor 36 may be taken out of any two numbers, as from 36 and 72, leaving 1 and 2. Now the other number may be multiplied by these factors.

Thus, $1\times2\times108=216$, the l. c. m. of 36, 72, and 108.

NOTE.—This method is not applicable unless the resulting quotients are prime to one another, but it is often found convenient where the numbers are large.

When the l. c. m. of several numbers is required, the following RULE is convenient:

Write the numbers in a horizontal line, divide by any prime number that will divide two or more of them without a remainder, and place the quotients and undivided numbers in a line below. Divide this line as before and thus proceed until no two numbers have a common factor greater than 1. The continued product of the divisors and numbers in the last line will be the l. c. m. required.

Example.—Find the l. c. m. of 8, 14, and 21.

OPERATION.

$$\begin{array}{r|rrr} 2 & 8 & 14 & 21 \\ \hline 7 & 4 & 7 & 21 \\ \hline & 4 & 1 & 3 \end{array} \qquad 2\times7\times4\times3=168$$

The pupil should verify the rule.

Exercise VII

By factoring, find the l. c. m. of:

1. 120, 144, and 200.
2. 11, 770, and 70.
3. 30, 90, and 180.
4. 95, 25, 285, and 625.

Using the highest common factor, find the l. c. m. of:

5. 804 and 536.
6. 248 and 372.
7. 96, 144, and 240.
8. 46, 115, and 161.

Using the rule, find the l. c. m. of:

9. 100, 240, and 215.
10. 27, 36, and 45.
11. 22, 33, 44, and 66.
12. 18, 27, 36, and 81.

REVIEW

1. Define arithmetic, number, and unit.

2. What is a composite number? A prime number?

3. What is a factor? A multiple?

4. What is the h. c. f. of two numbers? The l. c. m. of two numbers? Give examples.

5. What is the product of the h. c. f. and the l. c. m. of 12 and 15?

6. What numbers are exactly divisible by 3? By 4? By 9? By 11?

7. Show that 1188 is divisible by 2, 3, 4, 6, 9, and 11, but not by 5, 7, or 8.

8. Is 11 a factor of 3535? Of 445? Of 96745? Of 7543173? Of 1087362?

9. If A can build 14 rods of fence in a day, B 25 rods, C 8 rods, and D 20 rods, what is the least number of rods that will furnish a number of whole days' work to either one of the four men?

10. A street 399 ft. long and 35 ft. wide is to be paved with square flagstones of equal size, and as large as possible. How long and wide must each flagstone be?

11. Three ships arrive at a certain port, the first every Monday, the second every ten days, and the third every 12 days; they all arrived on Monday, May 1. When will all three arrive together again?

12. A man having a triangular piece of land, the sides of which are 165 ft., 231 ft., and 385 ft., wishes to inclose it with a fence having panels of the greatest possible uniform length; what must be the length of each panel?

CHAPTER IV

FRACTIONS

I. COMMON FRACTIONS

98. A **fraction** (Latin *frangere,* to break) is one or more of the equal parts of a unit.

Two units are involved in a fraction, the *unit of the fraction* and the *fractional unit.*

99. The **unit of the fraction** is 1, or the *one thing* divided into equal parts.

100. The **fractional unit** is one of the equal parts into which the *unit of the fraction* has been divided.

In the fraction $\frac{2}{3}$ bu., the *fractional unit* is $\frac{1}{3}$ bu., and the *unit of the fraction* is 1 bu.

101. A **common fraction** is a fraction expressed by two numbers, one above and one below a horizontal line; as $\frac{3}{5}$, which is read *three fifths.*

102. The **terms** of a fraction are the *numerator* and the *denominator.*

103. The **numerator** (Latin *numerare,* to number) is the number above the line. It shows the *number* of fractional units taken.

104. The **denominator** (Latin *denominare,* to name) is the number below the line. It shows the number of parts into which the unit of the fraction has been divided.

105. A **proper fraction** is one whose numerator is less than its denominator; as, $\frac{3}{5}$.

106. An **improper fraction** is one whose numerator is equal to, or greater than, its denominator; as, $\frac{7}{3}$.

107. A **mixed number** is a combination of an integer and a fraction; as, $4\frac{2}{3}$.

108. A **simple fraction** is a single fraction whose terms are integral numbers; as, $\frac{5}{8}$.

109. A **complex fraction** has one or both of its terms fractional; as, $\dfrac{3}{7\frac{1}{8}}, \dfrac{3\frac{2}{3}}{4}, \dfrac{\frac{2}{3}}{\frac{1}{8}}$.

110. **Similar fractions** are those that have like denominators; as, $\frac{4}{7}$ and $\frac{5}{7}$.

111. The **reciprocal** of a number is 1 divided by that number; thus the reciprocal of 8 is $\frac{1}{8}$.

112. The **three interpretations** of a fraction are illustrated by the following:

$\frac{3}{8}$ represents

1. 3 of the *eight equal parts* into which the unit of the fraction has been divided.
2. $\frac{1}{8}$ of 3 integral units.
3. 3 divided by 8.

113. **Fundamental principles** involved in fractions:

1. Multiplying the numerator of a fraction by any number multiplies the value of the fraction by the same number.

Reason.—This increases the number of parts without changing their size.

2. Multiplying the denominator of a fraction by any

number divides the value of the fraction by the same number.

Reason.—This decreases the size of the parts without changing the number of parts.

3. Multiplying both terms of a fraction by the same number does not change the value of the fraction.

Reason.—This increases the number of parts and decreases the size of the parts in the same ratio.

4. Dividing the numerator of a fraction by any number divides the value of the fraction by the same number.

Reason.—This decreases the number of parts without changing their size.

5. Dividing the denominator of a fraction by any number multiplies the value of the fraction by the same number.

Reason.—This increases the size of the parts without changing the number of parts.

6. Dividing both terms of a fraction by any number does not change the value of the fraction.

Reason.—This decreases the number of parts and increases the size of the parts in the same ratio.

REDUCTION OF FRACTIONS

114. The foregoing principles should be *consciously* applied by the learner in the various operations involving fractions.

Example.—1. Reduce $\frac{36}{48}$ to its lowest terms.

1ST SOLUTION: $\frac{36}{48} = \frac{\cancel{2} \times \cancel{2} \times \cancel{3} \times 3}{\cancel{2} \times \cancel{2} \times 2 \times 2 \times \cancel{3}} = \frac{3}{4}$. (Prin. 6, above.)

2D SOLUTION: $\frac{36}{48}=\frac{36 \div 12}{48 \div 12}=\frac{3}{4}$. (Prin. 6, above.)

Example.—2. Reduce $4\frac{3}{5}$ to an improper fraction.

SOLUTION: 1. $1=\frac{5}{5}$.
2. $4=4 \times \frac{5}{5}=\frac{20}{5}$. (Prin. 1, above.)
3. $\frac{20}{5}+\frac{3}{5}=\frac{23}{5}$.
∴ in $4\frac{3}{5}$ there are $\frac{23}{5}$.

Example.—3. Reduce $\frac{39}{4}$ to a whole or mixed number.

SOLUTION: $\frac{39}{4}=\frac{39 \div 4}{4 \div 4}=\frac{9\frac{3}{4}}{1}=9\frac{3}{4}$. (Prin. 6, above.)

ADDITION OF FRACTIONS

115. RULE.—**To add two or more fractions:**

STEPS:
1. *Reduce them to similar fractions.*
2. *Add their numerators.*
3. *Place the sum of the numerators over the denominator of the similar fractions.*

Example.—Add $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{1}{8}$, and $\frac{7}{10}$.

SOLUTION:

I. 1. The l. c. m. of the denominators is the *least common denominator.*
2. Therefore, the l. c. d. of $\frac{1}{2}$, $\frac{3}{4}$, $\frac{3}{5}$, $\frac{1}{8}$, and $\frac{7}{10}=40$.

II. 1. $\frac{1}{2}=\frac{1 \times 20}{2 \times 20}=\frac{20}{40}$.
2. $\frac{3}{4}=\frac{3 \times 10}{4 \times 10}=\frac{30}{40}$.
3. $\frac{3}{5}=\frac{3 \times 8}{5 \times 8}=\frac{24}{40}$.
4. $\frac{1}{8}=\frac{1 \times 5}{8 \times 5}=\frac{5}{40}$.
5. $\frac{7}{10}=\frac{7 \times 4}{10 \times 4}=\frac{28}{40}$.

III. $\frac{20}{40}+\frac{30}{40}+\frac{24}{40}+\frac{5}{40}+\frac{28}{40}=\frac{20+30+24+5+28}{40}=\frac{107}{40}.$

$\therefore \frac{1}{2}+\frac{3}{4}+\frac{3}{5}+\frac{1}{8}+\frac{7}{10}=\frac{107}{40}=2\frac{27}{40}.$

Exercise VIII

Find the sum of the following:

1. $\frac{14}{15}$, $\frac{17}{30}$, $\frac{23}{28}$, $\frac{19}{20}$.
2. $\frac{7}{12}$, $\frac{19}{24}$, $\frac{31}{36}$, $\frac{9}{10}$.
3. $\frac{2}{3}$, $\frac{5}{12}$, $\frac{17}{24}$, $\frac{11}{14}$.
4. $41\frac{10}{21}$, $23\frac{17}{42}$, $36\frac{13}{15}$.
5. $35\frac{17}{18}$, $26\frac{11}{12}$, $84\frac{15}{18}$.
6. $43\frac{18}{35}$, $19\frac{3}{10}$, $21\frac{11}{15}$.

SUBTRACTION OF FRACTIONS

116. Rule.—**To subtract one fraction from another:**

Steps:
1. *Reduce them to similar fractions.*
2. *Take the difference of their numerators.*
3. *Place the difference over the l. c. d.*

Example.—Subtract $\frac{2}{3}$ from $\frac{7}{8}$.

Solution:
1. The l. c. m. of 3 and 8 is 24.
2. $\frac{2}{3}=\frac{2\times 8}{3\times 8}=\frac{16}{24}.$ (Prin. 3, 113).
3. $\frac{7}{8}=\frac{7\times 3}{8\times 3}=\frac{21}{24}.$ " "
4. $\frac{21}{24}-\frac{16}{24}=\frac{21-16}{24}=\frac{5}{24}.$
5. $\therefore \frac{2}{3}$ from $\frac{7}{8}$ leaves $\frac{5}{24}$.

Exercise IX

Find the value of:

1. $\frac{65}{77}-\frac{45}{88}$.
2. $\frac{63}{75}-\frac{36}{50}$.
3. $\frac{83}{92}-\frac{30}{46}$.
4. $5\frac{1}{4}-3\frac{1}{3}$.
5. $9\frac{3}{16}-3\frac{13}{50}$.
6. $8\frac{1}{3}-4\frac{3}{4}$.
7. $8\frac{1}{9}-5\frac{7}{12}$.
8. $7\frac{5}{8}-2\frac{13}{16}$.
9. $9-3\frac{7}{16}$.

MULTIPLICATION OF FRACTIONS

117. RULE.—**To multiply one fraction by another:**

STEPS:

1. *Find the product of the numerators.*
2. *Find the product of the denominators.*
3. *Place the product of the numerators above the line for the numerator, and the product of the denominators below the line for the denominator.*

Example.—Multiply $\frac{3}{4}$ by $\frac{5}{7}$.

SOLUTION:

1. $1 \times \frac{3}{4} = \frac{3}{4}$.
2. $\frac{1}{7} \times \frac{3}{4} = \frac{1}{7}$ of $\frac{3}{4} = \frac{3}{7 \times 4}$. (Prin. 2, 113).
3. $\frac{5}{7} \times \frac{3}{4} = 5 \times \frac{3}{7 \times 4} = \frac{5 \times 3}{7 \times 4}$. (Prin. 1, 113).

$\therefore \frac{5}{7} \times \frac{3}{4} = \frac{5 \times 3}{7 \times 4} = \frac{15}{28}$.

NOTE.—When multiplying an integer by a fraction, or a fraction by an integer, the integer may be treated as a fraction by placing it above the line and putting 1 below the line for the denominator; thus, $6 = \frac{6}{1}$. Mixed numbers may be changed to improper fractions.

Exercise X

Find the product of:

1. $\frac{30}{49} \times \frac{7}{10}$.
2. $\frac{25}{48} \times \frac{8}{10}$.
3. $\frac{5}{18} \times \frac{6}{5}$.
4. $\frac{20}{33} \times \frac{10}{30}$.
5. $\frac{21}{16} \times \frac{12}{14}$.
6. $\frac{42}{50} \times \frac{10}{14}$.
7. $12\frac{3}{5} \times 15$.
8. $29\frac{8}{13} \times 26$.
9. $24\frac{15}{32} \times 48$.
10. $40 \times 4\frac{5}{8} \times 3\frac{2}{5}$.
11. $55 \times 6\frac{3}{20} \times 6\frac{7}{11}$.
12. $49 \times 2\frac{5}{6} \times 4\frac{4}{7}$.
13. $45 \times 8\frac{4}{5} \times 7\frac{5}{12}$.
14. $3\frac{1}{2} \times 4\frac{2}{3} \times 5\frac{3}{5} \times \frac{2}{9}$ of $\frac{5}{14} \times 6\frac{3}{4}$.

DIVISION OF FRACTIONS

118. Rule.—**To divide one fraction by another:**

First Method: *Reduce the fractions to similar fractions and divide the numerator of the dividend by the numerator of the divisor.*

Second Method: *Invert the divisor and proceed as in multiplication.*

Example.—Divide $\frac{3}{4}$ by $\frac{1}{6}$.

Solution by the First Method:

1. The l. c. m. of 4 and 6 is 12.
2. $\frac{3}{4}=\frac{3\times 3}{4\times 3}=\frac{9}{12}$.
3. $\frac{1}{6}=\frac{1\times 2}{6\times 2}=\frac{2}{12}$.
4. $\frac{9}{12}\div\frac{2}{12}=9\div 2=\frac{9}{2}$.
5. $\therefore\frac{3}{4}\div\frac{1}{6}=\frac{9}{2}=4\frac{1}{2}$.

Example.—Divide $\frac{2}{3}$ by $\frac{3}{4}$.

Solution by the Second Method:

1. $1\div\frac{1}{4}=4$.
2. $1\div\frac{3}{4}=\frac{1}{3}$ of $4=\frac{4}{3}$.
3. $\frac{2}{3}\div\frac{3}{4}=\frac{2}{3}\times\frac{4}{3}=\frac{8}{9}$.

$\therefore$ —

In equation 2, it will be seen that in finding how often the divisor is contained in 1, the divisor is *inverted*. Equation 3 gives us the rule for dividing one fraction by another.

The *note* under multiplication of fractions applies in the division of fractions.

Exercise XI

Find the quotient of

1. $\frac{32}{45} \div \frac{4}{5}$.
2. $\frac{5}{18} \div \frac{3}{4}$.
3. $\frac{15}{28} \div \frac{3}{7}$.
4. $\frac{17}{35} \div \frac{5}{6}$.
5. $6 \div \frac{5}{12}$.
6. $\frac{7}{15} \div 18$.

Show that:

7. $\dfrac{2\frac{4}{11}}{2\frac{3}{5}} \div \dfrac{2\frac{7}{11}}{8\frac{7}{10}} = 3.$
8. $\dfrac{7\frac{3}{7}}{40\frac{5}{9}} \div \dfrac{17\frac{1}{3}}{73} = \frac{27}{35}.$

THE H. C. F. OF FRACTIONS

119. The h. c. f. of two or more fractions is the *greatest fraction* that is contained an integral number of times in each of them.

In dividing one fraction by another, we invert the divisor; hence, in order that the quotient may be an integral number, the *numerator of the divisor* must be a *factor* of the *numerator of the dividend,* and the *denominator of the divisor* must be a *multiple* of the *denominator of the dividend.*

FORMULA.—The h. c. f. $= \dfrac{\text{the h. c. f. of the numerators.}}{\text{the l. c. m. of the denominators.}}$

120. RULE.—**To find the h. c. f. of two or more fractions:**

STEPS:
1. *Find the h. c. f. of the numerators.*
2. *Find the l. c. m. of the denominators.*
3. *Divide the first result by the second.*

Example.—Find the h. c. f. of $\frac{3}{5}$, $\frac{6}{7}$, $\frac{9}{14}$, and $\frac{12}{35}$.

SOLUTION:

1. The h. c. f. of 3, 6, 9, and 12 is 3.
2. The l. c. m. of 5, 7, 14, and 35 is 70.
3. 3 divided by 70 = $\frac{3}{70}$.
∴ The h. c. f. of $\frac{3}{5}$, $\frac{6}{7}$, $\frac{9}{14}$, and $\frac{12}{35}$ is $\frac{3}{70}$.

Exercise XII

Find the h. c. f. of:

1. $\frac{2}{3}$, $\frac{2}{5}$, and $\frac{4}{15}$.
2. $\frac{1}{7}$, $\frac{3}{14}$, and $\frac{7}{28}$.
3. $\frac{5}{8}$, $\frac{25}{32}$, and $\frac{7}{12}$.
4. $\frac{8}{9}$, $\frac{16}{27}$, and $\frac{32}{45}$.

THE L. C. M. OF FRACTIONS

121. The **l. c. m. of two or more fractions** is the least number (either integral or fractional) that will contain each of them an integral number of times.

A fraction is the *least common multiple* of two or more fractions, if its numerator is the l. c. m. of the given numerators, and its denominator the h. c. f. of the given denominators.

FORMULA.—The l. c. m. $= \dfrac{\text{the l. c. m. of the numerators.}}{\text{the h. c. f. of the denominators.}}$

122. RULE.—**To find the l. c. m. of two or more fractions:**

STEPS:
1. *Find the l. c. m. of the numerators.*
2. *Find the h. c. f. of the denominators.*
3. *Divide the first result by the second.*

Example.—Find the l. c. m. of $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{9}{10}$.

SOLUTION:

1. The l. c. m. of 3, 7, and 9 is 63.
2. The h. c. f. of 4, 8, and 10 is 2.
3. $63 \div 2 = \frac{63}{2}$.
∴ The l. c. m. of $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{9}{10}$ is $\frac{63}{2}$.

Exercise XIII

Find the l. c. m. of:

1. $\frac{3}{4}$, $\frac{9}{16}$, and $\frac{25}{36}$.
2. $\frac{2}{3}$, $\frac{5}{6}$, and $\frac{5}{8}$.
3. $9\frac{3}{8}$, $6\frac{3}{10}$, and $1\frac{1}{2}$.
4. $3\frac{1}{3}$, $\frac{5}{12}$, and $12\frac{1}{2}$.

REVIEW

1. Add $3\frac{1}{2}$, $4\frac{7}{8}$, $9\frac{5}{12}$, and $\frac{15}{16}$.

2. Show that $\left(2\frac{3}{4} + \frac{5}{3} \text{ of } \frac{7}{3\frac{4}{5}} - \frac{1\frac{3}{8}}{2\frac{1}{3}}\right) \div 1\frac{77}{228} = 5.$

3. Explain the reduction of $6\frac{3}{4}$ to $\frac{27}{4}$.

4. Which is greater, $\frac{17}{19}$ or $\frac{34}{37}$?

5. A man owns a tract of land, the sides of which are $134\frac{3}{4}$, $128\frac{1}{3}$, and $115\frac{1}{2}$ feet long: how many rails of the greatest length possible will be needed to fence in straight lines, the fence to be six rails high, and the rails to lap 6 inches at each end?—R. N. H., p. 94.

6. Find the difference in feet between $1\frac{7}{8}$ yards and $2\frac{2}{3}$ yards.

7. If the price of a duck is $\frac{5}{16}$ of the price of a turkey, and ducks sell for $1.25 a pair, find the price of a turkey.

8. A can walk around an island in $14\frac{2}{7}$ hours; B, in $9\frac{1}{11}$ hours; C, in $16\frac{2}{3}$ hours, and D, in 25 hours. If they start from the same point, and at the same time, how many hours after starting till they are all together again? —R. N. H., p. 95.

9. A can do a piece of work in 8 days, A and B can do it in 5 days, and B and C in 6 days. In what time can A, B and C do the work?

10. A fox has 100 yards the start of a hound. If the hound runs $12\frac{1}{2}$ yards while the fox runs 10 yards, how far will the hound run before he overtakes the fox?

II. DECIMAL FRACTIONS

123. **A decimal fraction** (Latin *decem,* ten) is a fraction whose denominator is some power of 10.

It is customary to express a decimal fraction by the use of the decimal point; thus,

$$\tfrac{7}{10}=.7;\ \tfrac{7}{100}=.07;\ \tfrac{27}{10000}=.0027.$$

When the denominator of a decimal fraction is expressed, the fraction may be called either a common fraction or a decimal fraction. Usually, however, the term decimal is used to mean a fraction whose denominator is indicated by the decimal point.

NOTE.—The first general use of the decimal point is found in the trigonometric tables of Pitiscus (1612).
—The New International Encyclopædia.

124. The **reading** for the decimal point is *and*. Thus, 3.47 is read 3 *and* 47 hundredths.

125. A **pure decimal** is one containing neither an integral part nor a common fraction; as, .342.

126. A **mixed decimal** is one containing an integral part; as, 42.75.

127. A **complex decimal** is one that ends with a common fraction; as, .33⅓.

128. A **finite** or **terminating decimal** is one that terminates, or comes to an end; as, .4½ = .45.

129. An **infinite** or **circulating decimal** is one that does not terminate; as, .7⅓ = .7333 . . . . to infinity.

130. Fundamental principles involved in decimals:

1. Moving the decimal point to the right *multiplies* the number by 10 once for each place the point is moved. Thus, .023 × 10 = .23.

2. Moving the decimal point to the left *divides* the number by 10 once for each place the point is moved. Thus, .023 ÷ 10 = .0023.

3. Annexing zeros to a decimal changes the *name,* but *not the value,* of the decimal. Thus, .423 = .4230 = .42300.

NOTE.—Show that the principles above are the same as principles 5, 2 and 3, 113.

131. Addition and **subtraction** of decimals offer no difficulties except such as are met in whole numbers. The processes are the same as those in whole numbers.

MULTIPLICATION OF DECIMALS

132. RULE.—**To multiply one decimal by another:**

STEPS:
1. *Multiply as in whole numbers.*
2. *Point off in the product as many places for decimals as there are decimal places in both multiplicand and multiplier.*

Example.—Multiply .24 by .05.

SOLUTION: $.24 \times .05 = .0120 = .012.$

PROOF: $$.24 \times .05 = \frac{24}{100} \times \frac{5}{100} = \frac{120}{10000} = .012.$$

DIVISION OF DECIMALS

133. RULE.—**To divide one decimal by another**

STEPS:
1. *Divide as in whole numbers.*
2. *Point off in the quotient as many places for decimals as the decimal places in the dividend exceed those in the divisor.*

Example.—Divide .144 by .04.

SOLUTION: $.144 \div .04 = 3.6.$

$$\text{PROOF: } .144 \div .04 = \frac{144}{1000} \div \frac{4}{100} = \frac{\overset{36}{\cancel{144}}}{\underset{10}{\cancel{1000}}} \times \frac{\cancel{100}}{\cancel{4}} = \frac{36}{10} = 3.6.$$

NOTE.—If the divisor has more decimal places than the dividend, annex zeros to the dividend until the number of decimal places in each is the same.

134. RULE.—**To change a decimal to a common fraction:** *Write the denominator under the numerator and reduce to the lowest terms.*

Thus, $.5 = \frac{5}{10} = \frac{1}{2}$; $.04 = \frac{4}{100} = \frac{1}{25}$.

135. RULE.—**To change a common fraction to a decimal:** *Divide the numerator by the denominator.*

Thus, $\frac{3}{4} = 3 \div 4 = 3.00 \div 4 = .75.$

$$\text{ANALYSIS: } \begin{cases} \frac{4}{4} = 100 \text{ hundredths} = 1.00. \\ \frac{1}{4} = \frac{1}{4} \text{ of } 1.00 = .25. \\ \frac{3}{4} = 3 \times .25 = .75. \end{cases}$$

CIRCULATING DECIMALS

136. Fundamental principles involved:

1. Any common fraction whose denominator contains no factor except 2 or 5 will give rise to a *finite* (i.e., *terminating*) decimal.

   Thus, $\frac{4}{25} = .16$; $\frac{3}{8} = .375$.

NOTE.—By "common fraction" here is meant a proper fraction in its lowest terms.

2. Any common fraction whose denominator does not

contain the factors 2 or 5, but factors prime to these, gives rise to a *pure* circulate.

Thus, $\frac{1}{3} = .333 \ldots$ to infinity.

3. Any common fraction whose denominator contains either of the factors 2 or 5, *and* some other factor prime to these, will give rise to a *mixed* circulate.

Thus, $\frac{4}{15} = .26666 \ldots$ to infinity.

Where a group of figures circulates, or recurs, a dot is placed over the first and the last figure of the group.

Thus, $\frac{1}{3} = .333 \ldots = .\dot{3}$; $\frac{42}{333} = .126126 \ldots = .\dot{1}2\dot{6}$.

The figure or group of figures continually repeated is called the **repetend.**

137. RULE.—**To change a pure circulate to a common fraction:**

STEPS:

1. *Write the figures of the circulate for the numerator.*
2. *Write as many 9's as there are figures in the circulate for the denominator.*
3. *Reduce to lowest terms.*

Example 1.—Change $.\dot{3}$ to a common fraction.

$$\text{SOLUTION: } \left\{ \begin{array}{ll} 1. & 10 \times .\dot{3} = 3.\dot{3} \\ 2. & \underline{1 \times .\dot{3} = .\dot{3}} \\ 3. & 9 \times .\dot{3} = 3. \\ 4. & .\dot{3} = \frac{3}{9}. \\ \therefore & .\dot{3} = \frac{3}{9} = \frac{1}{3}. \end{array} \right. \qquad \text{PROCESS: } .\dot{3} = \frac{3}{9} = \frac{1}{3}.$$

Example 2.—Change $.\dot{1}2\dot{6}$ to a common fraction.

$$\text{SOLUTION: } \left\{ \begin{array}{ll} 1. & 1000 \times .\dot{1}2\dot{6} = 126.\dot{1}2\dot{6} \\ 2. & \underline{1 \times .\dot{1}2\dot{6} = .\dot{1}2\dot{6}} \\ 3. & 999 \times .\dot{1}2\dot{6} = 126. \\ 4. & .\dot{1}2\dot{6} = \frac{126}{999}. \\ \therefore & .\dot{1}2\dot{6} = \frac{126}{999} = \frac{14}{111}. \end{array} \right. \qquad \begin{array}{c} \text{PROCESS:} \\ .\dot{1}2\dot{6} = \frac{126}{999} = \frac{14}{111}. \end{array}$$

138. Rule.—**To change a mixed circulate to a common fraction:**

Steps:
1. *Subtract the finite portion from the whole decimal (as in simple numbers), and write the result for a numerator.*
2. *For the denominator, write as many 9's as there are figures in the circulate, and annex as many 0's as there are finite places.*
3. *Reduce to lowest terms.*

Example 1.—Reduce $.64\dot{7}$ to a common fraction.

Solution:

1. $1000 \times .64\dot{7} = 647.\dot{7}$
2. $100 \times .64\dot{7} = 64.\dot{7}$
3. $900 \times .64\dot{7} = 583.$

$\therefore \quad .64\dot{7} = \frac{583}{900}.$

Process:

$$.64\dot{7} = \frac{647-64}{900} = \frac{583}{900}.$$

Example 2.—Change $.13\dot{8}$ to a common fraction.

Solution:

1. $1000 \times .13\dot{8} = 138.\dot{8}$
2. $100 \times .13\dot{8} = 13.\dot{8}$
3. $900 \times .13\dot{8} = 125.$

$\therefore \quad .13\dot{8} = \frac{125}{900} = \frac{5}{36}.$

Process:

$$\frac{138-13}{900} = \frac{125}{900} = \frac{5}{36}$$

139. Circulating decimals may be added, subtracted, multiplied, or divided. The simplest way of performing these operations is to change the circulates to equivalent common fractions; perform the indicated operation; then change the result to the decimal form.

Exercise XIV

1. Add .427, .3705, and .0727.
2. Add 1.33⅓ and 2.66⅔.

3. Subtract 72.0001 from 72.01.

4. How much is 19 – 8.999$\frac{1}{3}$?

5. Take 24$\frac{1}{2}$ tenths from 3701 thousandths.

6. Take 64$\frac{1}{6}$ hundredths from 100 units.

7. Multiply 42 units by 42 tenths.

8. Multiply 64.01 by .32.

9. Multiply 26 millions by 26 millionths.

10. Multiply 3.62741 by 1.6432, preserving two decimal places.

11 Multiply 432.5672 by 1.0666666, retaining 3 decimal places in the product.

12. Divide .36 by .008.

13. Divide *ten* by *one tenth.*

14. The product of four numbers is 932.25; three of them are 56.5, 1.1, and .03. What is the fourth?

15. Divide 487.24 by 1.003675, extending the quotient to two decimal places.

16. Divide 2 by 1.4142136, extending the quotient to three decimal places.

17. Reduce $.\dot{1}2\dot{3}$ to a common fraction.

18. Reduce $.208\dot{3}$ to a common fraction.

19. Reduce $.0902\dot{7}$ to a common fraction.

20. Reduce $.\dot{9}2307\dot{6}$ to a common fraction.

21. Multiply $.0\dot{4}3\dot{2}$ by 18.

22. Divide $.\dot{4}85\dot{7}$ by $.\dot{3}\dot{7}$.

REVIEW

1. Define fraction, fractional unit, unit of a fraction, and similar fractions.

2. What is the reciprocal of a number? Illustrate.

3. Give and illustrate the three interpretations of $\frac{5}{6}$ bu.

4. Give and illustrate the six fundamental principles of common fractions.

5. Explain the reduction of $3\frac{4}{5}$ to $\frac{19}{5}$.

6. Explain the reduction of $\frac{16}{7}$ to $2\frac{2}{7}$.

7. Add $\frac{2}{5}$, $\frac{4}{7}$, $\frac{7}{15}$, and $\frac{9}{35}$. Explain the process.

8. Subtract $\frac{5}{9}$ from $\frac{14}{15}$. Explain the process.

9. Multiply $\frac{3}{7}$ by $\frac{14}{15}$. Explain the process.

10. Divide $\frac{2}{5}$ by $\frac{3}{7}$. Explain the process.

11. Find the h. c. f. of $\frac{7}{9}$, $\frac{14}{27}$, and $\frac{28}{45}$. Explain the process.

12. Find the h. c. f. of $3\frac{1}{5}$, $1\frac{5}{7}$, and $\frac{24}{35}$.

13. Find the l. c. m. of $\frac{2}{5}$, $\frac{7}{10}$, $\frac{14}{15}$, and $\frac{8}{25}$.

14. Find the l. c. m. of $\frac{7}{24}$, $\frac{35}{36}$, and $\frac{49}{60}$.

15. Show that the sum of $\dfrac{\frac{2}{3} \text{ of } \frac{3}{4}}{\frac{1}{2}}$ and $\dfrac{\frac{2}{5} \text{ of } \frac{5}{6}}{\frac{2}{9} \text{ of } 4\frac{1}{2}}$ is equal to two times their difference.

16. A man after spending $\frac{1}{3}$, $\frac{2}{5}$, and $\frac{1}{8}$ of his money had $119 left. How much had he at first?

17. A post stands $\frac{1}{6}$ in the mud, $\frac{1}{4}$ in the water, and 21 feet above the water. What is its length?

18. What is the value of $17\frac{1}{2}$ bales of cotton, each weighing 475 pounds, at $9\frac{3}{8}$ cents per pound?

19. A boat whose rate of sailing in still water is 14 miles an hour, was accelerated $3\frac{1}{2}$ miles per hour in going down stream, and retarded the same distance per hour in coming up. How long would it take the boat to come up the same distance that it could go down in 10 hours?—M. S., p. 143.

20. How much is *a third and a half of a third* of 2?

21. One pipe can fill a tank in 5 hours; another in 8 hours, and another in 10 hours. How long would it take all together to fill the tank?

22. John is 18 years old, and $\frac{3}{4}$ of Henry's age is equal to $\frac{5}{6}$ of John's age. How old is Henry?

CHAPTER V

POWERS AND ROOTS

140. A **power** of a number is the product resulting from using the same number repeatedly as a factor.

Thus, 8 is a power of 2, since $2 \times 2 \times 2 = 8$.

141. The **powers are named** from the number of times the number is used as a factor.

Thus, $2 \times 2 = 4$, hence 4 is the *second power* of 2; $2 \times 2 \times 2 = 8$, hence 8 is the *third power* of 2; and so on. The *number* itself is called the *first power.*

142. The **square** of a number is its *second* power.

143. The **cube** of a number is its *third* power.

144. An **exponent** is a small figure or other character placed *to the right and above* a number to indicate the power required.

Thus, 3^2 means that the *second* power of 3 is required, or that 3 is to be used *twice* as a factor; *i.e.*, $3^2 = 3 \times 3 = 9$.

145. A **fractional exponent** denotes a *root*, the *denominator* being the *index* of the root, and the *numerator* denoting the *power of the number.*

Thus, $8^{\frac{1}{3}} = \sqrt[3]{8}$; $8^{\frac{2}{3}} = \sqrt[3]{8^2} = \sqrt[3]{64} = 4$.

I. INVOLUTION

146. The process of raising a number to a certain power is called **involution**

147. The term **involution** (Latin *involvere,* to roll up) signifies, in mathematics, the continued multiplication of any number by itself.

148. Fundamental principles of involution:

1. The square of any number contains *twice* as many figures, or *one less* than twice as many figures, as the number itself contains.
2. The cube of any number contains *three times* as many figures, or *one less* than three times, or *two less* than three times as many figures as the number itself contains.
3. The square of any number is equal to the square of its tens, plus twice the product of its tens and units, plus the square of its units.
   Thus, $(t+u)^2=t^2+2tu+u^2$.
4. The cube of any number is equal to the cube of its tens, plus three times the square of its tens by its units, plus three times its tens by the square of its units, plus the cube of the units.
   Thus, $(t+u)^3=t^3+3t^2u+3tu^2+u^3$.

149. Point out the four principles in the process of cubing 24:

$$
\begin{array}{rrr}
 & t + u & \qquad tu \\
 & 20 + 4 = & 24 \\
 & 20 + 4 = & 24 \\
\hline
 & (20 \times 4) + 4^2 = & 96 \\
 & 20^2 + (20 \times 4) \quad = & 48 \\
\hline
t^2 + 2tu + u^2 = & 20^2 + 2(20 \times 4) + 4^2 = & 576 \\
 & 20 + 4 = & 24 \\
\hline
 & (20^2 \times 4) + 2(20 \times 4^2) + 4^3 = & 2304 \\
 & 20^3 + 2(20^2 \times 4) + (20 \times 4^2) \quad = & 1152 \\
\hline
 & 20^3 + 3(20^2 \times 4) + 3(20 \times 4^2) + 4^3 = & 13824 \\
 & t^3 + 3t^2u \qquad + 3tu^2 \qquad + u^3. &
\end{array}
$$

Note.—For convenience, any number may be considered as made up of *tens* and *units*. Thus, 346 = 34 *tens* and 6 *units*. It is customary to let t represent the *tens* and u the *units*, as in the work above. The pupil should be able to show that the two operations above are identical.

150. Using the method indicated in principles 3 and 4 above, find the square and the cube of:

1. 34.
2. 46.
3. 51.
4. 68.
5. 265.
6. 387.
7. 989.
8. 625.

APPLICATION

151. The following principles of physics furnish application for the rules of involution:

1. The intensity of light varies inversely as the square of the distance from the source of the light.
2. The intensity of sound varies inversely as the square of the distance from the source of the sound.
3. The intensity of heat varies inversely as the square of the distance from the source of the heat.
4. Gravitation varies inversely as the square of the distance between the centers of gravity.

Exercise XV

1. A body on the earth's surface (4000 miles from the center of the earth) weighs 200 pounds. What would it weigh 1000 miles above the earth's surface?

2. A bell heard by A is heard by B five times as far away. How loud does it sound to A as compared to B?

3. If the earth were removed to $\frac{1}{4}$ its distance from the sun, how much more intense would be the heat received?

4. A has two electric lights, one of 4 candle power and one of 16 candle power. If the one of 4 candle power is 8 feet away, how far away must the other be placed to give a light of equal intensity?

II. EVOLUTION

152. Evolution (Latin *evolvere,* to unroll) is the process of finding roots of numbers. It is the *inverse* of involution.

153. A **root** is a factor repeated to produce a power.

154. The **radical sign** ($\sqrt{}$), when placed before a number, indicates that its root is to be extracted.

155. The **index** of the root is the figure placed above the radical sign to indicate what root is to be taken. When no index is written, the index two is always understood.

156. The **names of roots** are derived from the corresponding powers, and are indicated by the indices of the radical sign.

Thus, $\sqrt{16}$, or $\sqrt[2]{16}$, denotes the *square root* of 16; $\sqrt[3]{16}$, the *cube root* of 16; $\sqrt[4]{16}$, the *fourth root* of 16; etc.

The same may be denoted by fractional exponents.

Thus, $\sqrt{16}=16^{\frac{1}{2}}$; $\sqrt[3]{16}=16^{\frac{1}{3}}$; $\sqrt[4]{16}=16^{\frac{1}{4}}$; etc.

SQUARE ROOT

157. The **square root** of a number is one of the *two equal factors* that produce the number.

158. First Method.—The square root of an integral square can usually be found by factoring.

Example.—Find the square root of 324.

OPERATION:

2	324
2	162
3	81
3	27
3	9
	3

$$\text{Since } 324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$$
$$= 18 \times 18$$
$$\therefore \sqrt{324} = 18.$$

159. Second Method.—When numbers are large, or when ot perfect squares, the method to be used in extracting he square root is developed from the following equation:

$$(t + u)^2 = t^2 + 2tu + u^2 = t^2 + (2t + u)u.$$

RULE.—**To find the square root of a number:**

STEPS:

1. *Beginning with units, point off the number into periods of two figures each* (principle 1, **148**) *to find the number of figures in the root.*
2. *Find the greatest square in the left-hand period and write its root for the first figure in the root.*
3. *Subtract the square from the period, and to the remainder annex the next period for the dividend.*
4. *For a trial divisor, double the root found and annex one zero.*
5. *Divide, and write the quotient for the trial figure in the root.*
6. *Add the trial figure of the root to the trial divisor for a complete divisor.*
7. *Multiply the complete divisor by the trial term in the root, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*
8. *Repeat, beginning with 4th step.*

EXAMPLES

1. What is the square root of 15625?

Operation: $\dot{1}5\dot{6}2\dot{5}\,|\,\underline{125}$, *ans.*

	$1^2 =$	1	
Trial divisor	20	56	
Complete "	22	44	= product of complete divisor by 2.
Trial "	240	1225	
Complete "	245	1225 =	" " " " " 5.

2. What is the square root of 3906.25?

Operation: $3\dot{9}0\dot{6}.2\dot{5}\,|\,\underline{62.5}$, *ans.*

	$6^2 =$	36
Trial divisor	120	306
Complete "	122	244
Trial "	124.0	62.25
Complete "	124.5	62.25

Note.—The learner should study the examples above, together with the "steps". It would be well for the teacher to explain the operation along with this formula, $(t + u)^2 = t^2 + (2t + u)u$. This will be a very simple matter, if reference is made to **149.**

160. Rule.—**To find the square root of a common fraction:**

First Method: *Find the square root of each term of the common fraction.*

Second Method: *Reduce the common fraction to a decimal and proceed as in whole numbers.*

EXAMPLES

1. Find the square root of $\frac{16}{25}$.

$\sqrt{16} = 4$; $\sqrt{25} = 5$; $\therefore \sqrt{\frac{16}{25}} = \frac{4}{5}$

Proof: $\frac{4}{5} \times \frac{4}{5} = \frac{16}{25}$

2. Find the square root of $\frac{3}{7}$.

First Method: $\sqrt{\frac{3}{7}} = \sqrt{\frac{3 \times 7}{7 \times 7}} = \sqrt{\frac{1}{49} \times 21} = \frac{1}{7}\sqrt{21} = .654+.$

Second Method: $\frac{3}{7} = .428571+$; $\sqrt{.428571+} = .654+.$

Exercise XVI

Find the value of:

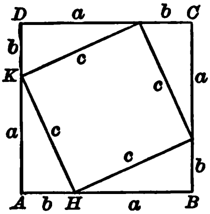
1. $\sqrt{1024}$
2. $\sqrt[2]{1225}$
3. $\sqrt{\frac{6}{7}}$
4. $\sqrt{2025}$
5. $14400^{\frac{1}{2}}$
6. $\sqrt{118.81}$
7. $\sqrt{185.640625}$
8. $\sqrt{272\frac{3}{12}}$
9. $\sqrt{32 \times 2.}$

APPLICATION OF SQUARE ROOT

The Pythagorean Theorem.—*In a right-angled triangle the sum of the squares on the two sides equals the square on the hypotenuse.*

Note.—This is called the Pythagorean theorem, because it is supposed to have been first proved by Pythagoras about 550 B.C.

Demonstration.—In the right-angled triangle *AHK*, let the sides be represented by *a*, *b*, *c*.



1. $(a + b)^2 = a^2 + b^2 + 2ab$ (148, 3).
2. $(a + b)^2 = c^2 +$ the 4 triangles.
3. $(a + b)^2 = c^2 + 4 \times \frac{1}{2}ab = c^2 + 2ab$.
4. $a^2 + b^2 + 2ab = c^2 + 2ab$ (14, 9).

$\therefore$ $a^2 + b^2 = c^2$.—Q. E. D.

10. Two vessels sail from the same port, one due north 6 miles an hour, and the other due east 8 miles an hour: how far apart are they at the end of 12 hours?

11. A ladder 39 feet long, when the foot is placed 15 feet from a wall, reaches the wall four feet below the top; how high is the wall?

12. A room is 20 feet long, 16 feet wide, and 12 feet high. What is the distance from one of the lower corners to the opposite upper corner?

13. The sides of two square blocks are 5 feet and 10 feet, respectively. How do they compare in area?

14. A pendulum which is 65 inches long swings from a point A to a point B, and A and B are in a horizontal line. If the line AB is four inches above the lowest point in the sweep, how far is A from B?

15. In extracting the square root of a perfect power, the last complete dividend was found 4725: what was the power?—R. N. H., p. 367.

16. A hall, standing east and west, is 46 ft. by 22 ft., and $12\frac{1}{2}$ ft. high: what is the length of the shortest path a fly can travel, by walls and floor, from a southeast lower corner to a northwest upper corner?—R. N. H., p. 367.

CUBE ROOT

161. The **cube root** of a number is one of the *three equal factors* that produce the number.

162. First Method.—The cube root of an integral cube can usually be found by factoring.

Example.—Find the cube root of 3375.

OPERATION:

$$\begin{array}{r|r} 3 & 3375 \\ \hline 3 & 1125 \\ \hline 3 & 375 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline & 5 \end{array}$$

$$\text{Since } 3375 = 3 \times 3 \times 3 \times 5 \times 5 \times 5.$$
$$= 15 \times 15 \times 15.$$
$$\therefore \sqrt[3]{3375} = 15.$$

163. Second Method.—When numbers are large, or when they are not perfect cubes, the method to be used in extracting the cube root is developed from the following equation:

$$(t + u)^3 = t^3 + 3\,t^2u + 3\,tu^2 + u^3 = t^3 + (3t^2 + 3\,tu + u^2)\,u.$$

RULE.—**To find the cube root of a number:**

STEPS:

1. *Beginning with the units, point off the number into periods of three figures each* (principle 2, **148**) *to find the number of figures in the root.*
2. *Find the greatest cube in the left-hand period and write its root for the first figure in the root.*
3. *Subtract the cube from the period, and to the remainder annex the next period for a dividend.*
4. *For a trial divisor, write three times the square of the root found, considered as tens.*
5. *Divide, and write the quotient for the trial figure in the root.*
6. *For the complete divisor, add to the trial divisor 3 times the product of the last trial figure by the preceding part of the root considered as tens, and to this sum add the square of the trial term.*
7. *Multiply the complete divisor by the trial term in the root, subtract the product from the dividend, and to the remainder annex the next period for a new dividend.*
8. *Repeat, beginning with the 4th step.*

EXAMPLES

1. What is the cube root of 13144256?

OPERATION.

		13144256	2 (1st root-figure).
$2^3 =$		8	
1st trial divisor $= 3 \times 20^2 =$	1200	5144	3 (2d root-figure).
To which add $3 \times 20 \times 3 =$	180		
To which add $3^2 =$	9		
1st complete divisor =	1389	4167	
2d trial divisor $= 3 \times 230^2 =$	158700	977256	6 (3d root-figure).
To which add $3 \times 230 \times 6 =$	4140		
To which add $6^2 =$	36		
2d complete divisor =	162876	977256	236 (root).

2. What is the cube root of 154854.153?

OPERATION.

		154854.153	5 (1st root-figure).
$5^3 =$		125	
1st trial divisor $= 3 \times 50^2 =$	7500	29854.	3 (2d root-figure).
To which add $3 \times 50 \times 3 =$	450		
To which add $3^2 =$	9		
1st complete divisor =	7959	23877.	
2d trial divisor $= 3 \times 53^2.0 =$	8427.00	5977.153	.7 (3d root-figure).
To which add $3 \times 53.0 \times .7 =$	111.30		
To which add $.7^2 =$	.49		
2d complete divisor =	8538.79	5977.153	53.7 (root).

NOTE.—The learner should study the examples above, together with the "steps." It would be well, also, to show that the following formula is an algebraic expression of the "steps":

$$(t + u)^3 = t^3 + 3t^2u + 3tu^2 + u^3 = t^3 + (3t^2 + 3tu + u^2)u.$$

164. RULE.—**To find the cube root of a fraction:**

FIRST METHOD: *Find the cube root of each term of the fraction.*

Second Method: *Reduce the fraction to a decimal, and proceed as in whole numbers.*

EXAMPLES

1. Find the cube root of $\frac{8}{27}$.

$$\sqrt[3]{8} = 2; \quad \sqrt[3]{27} = 3; \quad \therefore \sqrt[3]{\tfrac{8}{27}} = \tfrac{2}{3}.$$

Proof: $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27}$.

2. Find the cube root of $\frac{6}{7}$.

First Method: $\sqrt[3]{\tfrac{6}{7}} = \sqrt[3]{\tfrac{6}{7} \times \tfrac{7}{7} \times \tfrac{7}{7}} = \sqrt[3]{\tfrac{1}{343} \times 294}$
$= \frac{1}{7}\sqrt[3]{294} = .949 +.$

Second Method: $\frac{6}{7} = .857142 +; \ \sqrt[3]{.857142+} = .949+.$

Exercise XVII

Find the value of:

1. $\sqrt[3]{12167}$
2. $\sqrt[3]{34328125}$
3. $131096512^{\frac{1}{3}}$
4. $\sqrt[3]{633839.779}$
5. $\sqrt[3]{.000512}$
6. $\sqrt[3]{15.625}$
7. $\left(\frac{343}{12167}\right)^{\frac{1}{3}}$
8. $\sqrt[3]{\frac{64}{125}}$
9. $\sqrt[3]{.000008.}$

APPLICATION OF CUBE ROOT

10. A room is twice as long as high, and the width is equal to the height: what is the height if the capacity of the room is 3456 cubic feet?

11. A cubical box contains 474552 cubic inches. What is the area of one of its sides?

12. What is the entire surface of a cube whose volume is 91125 cu. ft.?

13. What is one dimension of a cubical bin that will hold

the same number of bushels of wheat as a bin that is $92\frac{4}{7}$ inches long, 56 inches wide, and 6 feet deep?

14. A rectangular reservoir with square base is 7 times as high as wide, and has a capacity of 1089 gallons. Find its dimensions.

15. A cubic inch of iron weighs 4.5 oz. A bar of iron, twice as wide as thick and three times as long as wide, weighs 27 pounds. Find its dimensions.

16. What will it cost to plaster the bottom and sides of a cubical reservoir which will hold 53361 gallons of water, at 4¢ a square foot?

CHAPTER VI

RATIO AND PROPORTION

I. RATIO

165. Ratio is the relation of one number to another of the same kind. It is expressed by the quotient of the first divided by the second.

Thus, the ratio of 3 to 4 is $\frac{3}{4}$.

166. The **sign of a ratio** is the colon (:), which is the sign of division with the horizontal line omitted.

Thus, $8 : 7$ signifies the ratio of 8 to $7 = \frac{8}{7}$.

167. The **terms** of a ratio are the two numbers compared, and are called *antecedent* and *consequent.* The two terms form a *couplet.*

Thus, $3 : 5$ is a couplet.

168. The **antecedent** (Latin *antecedere,* to go before) is the *first term,* or *dividend.*

169. The **consequent** (Latin *consequi,* to follow) is the *second term,* or *divisor.*

170. The **value** of a ratio is found by dividing the antecedent by the consequent, and it is always an abstract number.

171. A **simple ratio** is a ratio consisting of two terms; as, $6 : 5 = \frac{6}{5}$.

172. A **compound ratio** is a ratio whose terms are the continued products of the corresponding terms of two or more simple ratios; as,

$$\left\{\begin{matrix} 3 : 4 \\ 7 : 9 \end{matrix}\right\} = \frac{3 \times 7}{4 \times 9}.$$

173. A **direct ratio** is the quotient of the antecedent divided by the consequent.

174. A **reciprocal** or **inverse ratio** is the quotient of the consequent divided by the antecedent.

175. Fundamental principles of ratio:

1. The two terms of a ratio must be like numbers.
2. If the product of the two terms of a ratio be divided by either term, the quotient will be the other term.
3. The value of a ratio is not changed by multiplying or dividing both terms by the same number.
4. The product of two or more ratios equals the ratio of the product of their antecedents to the product of their consequents.

Thus, $(4 : 7) \times (8 : 9) = 4 \times 8 : 7 \times 9$.

Exercise XVIII

1. Find the ratio of 3 to 7.

2. Find the inverse ratio of 3 to 7.

3. What is the ratio of 12 bushels to 4 bushels?

4. Can there be any ratio between 3 feet and 6 bushels? (Give reason for your answer.)

5. Which is greater 3 : 4 or 8 : 9?

6. Reduce $\left\{\begin{matrix} 1\frac{2}{3} : 5 \\ 10 : 1\frac{3}{7} \end{matrix}\right\}$ to a simple ratio.

7. Which is greater \$2½ : \$5, or 3½ ft.: 6 ft.?

8. The antecedent is 15, the ratio ⅘; find the consequent.

9. The consequent is 6.12½, the ratio 25; find the antecedent.

10. The antecedent is ½ of ⅝ and the consequent is .75; find the ratio.

II. PROPORTION

176. Proportion (Latin *pro,* before, + *portio,* share) is an expression of equality of ratios.

177. The **sign of proportion** is the double colon (::).

NOTE.—The sign of equality (=) is often used instead of the double colon.

178. The **terms** in a proportion are the numbers that make up the proportion.

179. The **extremes** are the first and fourth terms.

180. The **means** are the second and third terms.

181. A **proportional** is any term of a proportion.

182. A **mean proportional** is a number which is used as the *consequent* in the first couplet, and as the *antecedent* in the second.

Thus, in 4 : 8 : : 8 : 16, 8 is a *mean proportional* between 4 and 16.

183. Fundamental principles of proportion are:

1. The product of the means is equal to the product of the extremes.

PROOF: If a:b::c:d, then $\frac{a}{b} = \frac{c}{d}$, and clearing of fractions, ad = bc.

2. A mean proportional is equal to the square root of the product of the two other terms.

Proof: If $a:b::b:c$, then $b^2 = ac$ (prin. 1).
Extracting the square root of each member, $b = \sqrt{ac}$.

3. The product of the means divided by either extreme gives the other extreme.

Proof: If $a:b::c:d$, then $ad = bc$, $\therefore a = \frac{bc}{d}$.

4. The product of the extremes divided by either mean gives the other mean.

Proof: If $a:b::c:d$, then $ad = bc$, $\therefore b = \frac{ad}{c}$.

SIMPLE PROPORTION

184. Simple proportion is an expression of equality between two simple ratios.

Note.—Formerly, proportion was called the **Rule of Three,** from the fact that *three numbers were given* to find a fourth. Simple proportion was called **Single Rule of Three,** and compound proportion, **Double Rule of Three.**

185. Statement.—Every problem in proportion consists of two parts, a *known part* and an *unknown part.*

1. Determine the known part.
2. Determine the 3d term, or base term.
3. Reason from the known to the unknown.

Example.—If four desks cost $20, what will 7 desks cost?

1. Known part: 4 desks, $20.
2. Unknown part: 7 " $x.

Operation:

(1) 4 desks: 7 desks : : $20: $x.

(2) $\therefore \$x = \frac{7 \times \$20}{4} = \$35.$ (Prin. 3, 183.)

EXPLANATION.—Write \$20 for the 3d term, since it is the same kind as is required in the result. If 4 desks cost \$20, 7 desks will cost *more* than \$20; therefore, write the greater number for the 2d term and the lesser for the 1st. The product of the means divided by one extreme gives the other extreme. We cannot multiply \$20 by 7 *desks;* therefore, we use the *ratio* of 4 desks to 7 desks, which is $\frac{4}{7}$ or 4:7. The terms of either couplet may thus be made abstract.

SOLUTION BY ANALYSIS:

1. Since the cost of 4 desks = \$20,
2. $\therefore$ " " " 1 desk = $\frac{1}{4}$ of \$20 = \$5,
3. and " " " 7 desks = 7 × \$5 = \$35.

Exercise XIX

Find the missing terms in the following:

1. 3 :9 : :7 :( ).
2. 45 :9 : :25 :( ).
3. 21 :3 : :( ) :5.
4. 64 :( ) : :( ) :1.
5. ( ) :10 : :8 :16.
6. $\frac{5}{2}$:( ) : :$8\frac{4}{7}$:$12\frac{3}{14}$.
7. $\frac{2}{3}$: $\frac{5}{6}$: :( ) :$\frac{6}{7}$.
8. 2.5 :.5 : :.25 :( ).

9. If $82\frac{1}{2}$ bushels of potatoes are raised on $\frac{33}{32}$ acres, how many bushels can be raised on $3\frac{1}{4}$ acres?

10. The Washington monument casts a shadow 223 ft. 6.5 in., when a post 3 ft. high casts a shadow 14.5 in. Find the height of the monument.

11. If the interest received on a certain sum of money for 1.5 yr. is \$27, how much is the interest on the same sum at the same rate for 2 mo.?

12. A lawyer who collects for 5% gets \$34.60 for collecting a debt. Find the amount of the debt.

13. A's property is assessed at \$3800. What is his tax at 96¢ on the \$100?

14. A man can do a certain piece of work in 18 days, working 8 hours a day. In how many days can he do the same work by working 10 hours a day?

15. If 36 yards of carpet $\frac{3}{4}$ of a yard wide will cover my office floor, how many yards $\frac{1}{2}$ of a yard wide will be required to cover it?

16. A man can dig a ditch in 6 days; he and his son can dig it in 4 days. In how many days can the son dig it?

17. If $\frac{5}{6}$ of the value of a ship is $11000, what is $\frac{2}{3}$ of its value?

18. The ratio of A's pay to B's pay is $\frac{2}{3}$. B's pay is $27 per week. What is A's pay per week?

COMPOUND PROPORTION

186. A **compound proportion** is a proportion which contains a compound ratio.

The method of reasoning and the principles given in simple proportion apply in compound proportion.

EXAMPLES

1. If 11 men can cut 147 cords of wood in 7 days, working 14 hours a day, how many days will be required for 5 men to cut 150 cords, working 10 hours a day?

1. Known: 11 men, 147 cords, 7 days, 14 hours.
2. Unknown: 5 men, 150 " x " 10 "

OPERATION:

$$\left.\begin{array}{r} 5 : 11 \\ 147 : 150 \\ 10 : 14 \end{array}\right\} : : 7 \text{ days} : x \text{ days.}$$

Or, $$5 \times 147 \times 10 : 11 \times 150 \times 14 : : 7 \text{ days} : x \text{ days.}$$

$$\therefore x \text{ days} = \frac{11 \times 150 \times 14 \times 7 \text{ days}}{5 \times 147 \times 10} = 22 \text{ days.}$$

Explanation.—7 days is written for the 3d term, because it is the same kind as is required in the result.

1. If 11 men require 7 days, 5 men will require *more* than 7 days; therefore, the greater number (11) is written for the 2d term, and the lesser (5) for the 1st.

2. If 147 cords require 7 days, 150 cords will require *more* than 7 days; therefore, the greater number (150) is written for the 2d term, and the lesser (147) for the 1st.

3. If 14 hours a day require 7 days, 10 hours a day will require *more* than 7 days; therefore, the greater number (14) is written for the 2d term, and the lesser (10) for the 1st.

Multiplying the 3d term by the continued product of the 2d, and dividing by the continued product of the 1st, gives 22 days.

Note.—Employ cancellation wherever possible.

SOLUTION BY ANALYSIS:

1. Since time for 11 men working 14 hr. a da. to cut 147 c.
= 7 da.,
2. the " " 1 man " 14 " " " to cut 147 c.
$= 11 \times 7$ da.
3. " " " 1 " " 1 " " " to cut 147 c.
$= 14 \times 11 \times 7$ da.
4. " " " 1 " " 1 " " " to cut 1 c.
$= \frac{14 \times 11 \times 7 \text{ da.}}{147}$
5. " " " 1 " " 1 " " " to cut 150 c.
$= \frac{150 \times 14 \times 11 \times 7 \text{ da.}}{147}$
6. " " " 1 " " 10 " a da. to cut 150 c.
$= \frac{150 \times 14 \times 11 \times 7 \text{ da.}}{10 \times 147}$
7. " " " 5 men " 10 " a da. to cut 150 c.
$= \frac{150 \times 14 \times 11 \times 7 \text{ da.}}{5 \times 10 \times 147} = 22$ da.

Note.—After the pupil becomes familiar with each step, he may pass from step 1 to step 4, and then to step 7. By waiting until the last step, numerous cancellations simplify the work.

2. If four men build a wall 72 ft. long, 5 ft. wide, and 2 ft. high in 12 days, how many men can build a wall 120 ft. long, 6 ft. wide, and $1\frac{1}{2}$ ft. high in 9 days?

1. **Known:** 4 men, 72 ft. l., 5 ft. w., and 2 ft. h., 12 da.
2. **Unknown:** x " 120 " " 6 " " " $1\frac{1}{2}$ " " 9 "

OPERATION:

$$\left.\begin{array}{r@{\;:\;}l} 72 & 120 \\ 5 & 6 \\ 2 & 1.5 \\ 9 & 12 \end{array}\right\} :: 4 \text{ men} : x \text{ men.}$$

$$\text{Or, } 72 \times 5 \times 2 \times 9 : 120 \times 6 \times 1.5 \times 12 :: 4 \text{ men} : x \text{ men.}$$

$$\therefore x \quad \text{men} = \frac{120 \times 6 \times 1.5 \times 12 \times 4 \text{ men}}{72 \times 5 \times 2 \times 9.} = 8 \text{ men.}$$

Study the example above until you can give the *reason* for the arrangement of each couplet.

Exercise XX

1. If a man travels 145 miles in 5 days, traveling 12 hours a day, how many days will be required for him to travel 435 miles, traveling 6 hours a day?

2. If 18 men, working 10 hours a day, finish a task in one day, how many days will it require 10 men to do the same work, by working 9 hours a day?

3. A contracted to build a wall 500 ft. long in 20 days; 20 men built 300 ft. in 15 days: how many additional men must be employed to complete the wall in the required time?

4. If a cistern $17\frac{1}{2}$ ft. long, $10\frac{1}{2}$ ft. wide, and 13 ft. deep holds 546 barrels, how many barrels will a cistern hold that is 16 ft. long, 7 ft. wide, and 15 ft. deep?

5. If 27 men can perform a piece of work in 19 days,

how many men can do another piece of work $\frac{7}{8}$ as great in $\frac{3}{8}$ of the time?

6. How many hours a day ought 30 men to labor to do in 10 days a piece of work which is $\frac{2}{3}$ as great as a similar job which 25 men, working 12 hours a day, did in 12 days?

7. If a bin 8 ft. long, 6 ft. wide, and 4.5 ft. deep, will hold 180 bushels of wheat, how deep must another bin be, that is 12 ft. long and 9 ft. wide, to hold 405 bushels?

8. A contractor engaged to pave 15 miles of a road in 12 months, and for that purpose employed 100 men. At the end of seven months he had completed only 6 miles. How many more men did he need to finish the work in the time prescribed?

9. In how many days will six persons consume 5 bushels of potatoes, if 3 bushels and 3 pecks last 9 persons 22 days?

10. If 18 men build 24 rods of fence in 12 days, working 7 hours a day, how many men will it take to build 80 rods in 14 days, working 6 hours a day?

187. Cause and Effect.—Problems in compound proportion may easily be solved by *cause and effect,* but this method reduces the problem to *simple* proportion and becomes rather mechanical. It does not develop the power to reason so well as does the method by compound proportion.

Problem number 10 will be solved as an illustration. The statement may be arranged thus:

1st cause :	2d cause : :	1st effect :	2d effect
18 men 12 days 7 hours	x men 14 days 6 hours	24 rods	80 rods

Considering all terms *abstract,* we have

$$x \times 14 \times 6 \times 24 = 18 \times 12 \times 7 \times 80.$$

$$\therefore x = \frac{18 \times 12 \times 7 \times 80}{14 \times 6 \times 24} = 60.$$

Ans. 60 men.

PARTITIVE PROPORTION

188. Partitive proportion is the process of dividing a number into *parts bearing a given relation to each other.*

EXAMPLES

1. Divide 21 into parts proportional to 3 and 4.

EXPLANATION.—Since the parts bear the ratio of 3 to 4, for every 7 (sum of 3 and 4) there is a 3 and a 4. Therefore, one part is $\frac{3}{7}$ of 21, or 9, and the other is $\frac{4}{7}$ of 21, or 12.

2. Divide $39 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

EXPLANATION.—Reducing the fractions to a common denominator, we have $\frac{6}{12}$, $\frac{4}{12}$, and $\frac{3}{12}$. Now the parts are proportional to 6, 4, and 3. Proceed as in example 1. The parts are $18, $12, and $9.

Exercise XXI

1. Divide 105 into parts proportional to 5, 7, and 9.

2. Three persons gain $2640, of which A is to receive $6 as often as B receives $4, and C $2. What is each one's share?

3. Divide $1596 into parts proportional to $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{4}{5}$.

4. If common salt is composed of 71 parts of chlorine to 46 parts of sodium, how many pounds of each element are there in 468 pounds of common salt?

5. A, B, and C bought a house for $9000, A furnishing $4500, B $2500, and C $2000. The house rents for $810; find each one's share of the rent.

CHAPTER VII

SERIES

189. A **series** (Latin *series*, a row) is a succession of numbers formed according to some common law.

190. The **terms** of a series are the numbers composing it.

191. The **extremes** are the first and last terms.

192. The **means** are the intermediate terms.

193. An **ascending series** is one in which each term is greater than the one preceding it.

194. A **descending series** is one in which each term is less than the one preceding it.

195. There are many kinds of series, but it is customary to consider only two kinds in arithmetic.

These two are called *progressions*.

I. ARITHMETICAL PROGRESSION

196. An **arithmetical progression** is a series of numbers increasing or decreasing by a common difference.

197. The **common difference** is the difference between any two adjacent terms.

198. In an arithmetical progression there are *five parts*, any *three* of which being given, the other *two* can be found. The five parts with which we are concerned are:

1. The first term, represented by a.
2. The last term, " " l.

3. The number of terms, represented by n.
4. The common difference, " " d.
5. The sum of all the terms, " " s.

199. The **two fundamental formulas** of arithmetical progression are:

I. The formula for the last term, $l = a + (n - 1)d$.

This formula is derived from a series as follows:

(1)	(2)	(3)	(4)	(5)	(6)	
a,	$a+d$,	$a+2d$,	$a+3d$,	$a+4d$,	$a+5d$,	etc.

From the series we observe that *any* term consists of the first term + the common difference taken as many times as there are terms *less* 1. Hence, for the nth term we have $a + (n-1)d$; or, letting l stand for the nth term, or last term, $l = a + (n-1)d$.

Study the formula and then *write the rule* for finding the *last term* in an arithmetical series.

From the above formula, derive the formula for finding the *first term*; the *number of terms*; the *common difference*. *Write the rule* corresponding to each formula.

In a descending series, d is subtracted and the formula is written $l = a - (n - 1)d$.

II. The formula for the sum of the series, $s = \frac{n}{2}(a + l)$.

This formula may be derived as follows:

Since the sum of a series is simply the sum of all the terms in the series, we may write,

$$
\begin{array}{rl}
 & s = a + (a + d) + (a + 2d) + \ldots\ldots (l - 2d) + (l - d) + l. \\
\text{or,} & \underline{s = l + (l - d) + (l - 2d) + \ldots\ldots (a + 2d) + (a + d) + a.} \\
\text{Adding,} & 2s = (a+l) + (a+l) + (a+l) + \ldots\ldots (a+l) + (a+l) + (a+l).
\end{array}
$$

$\therefore 2s = (a + l)$ taken as many times as there are terms (n).

$\therefore 2s = n(a + l)$, and $s = \frac{n}{2}(a + l) = n\frac{(a + l)}{2}$.

Study this formula, and then *write the rule* for finding the *sum* of an arithmetical series. From the formula above, derive the formula for finding the *first term;* the *number of terms;* the *last term. Write the rule* corresponding to each formula.

EXAMPLES

1. Find the 12th term of the series 2, 4, 6, 8, ...

SOLUTION: $l = a + (n - 1)d$; $\therefore l = 2 + (12 - 1)\,2 = 24$, *ans.*

2. Find the 14th term of the series 64, 61, 58, 55, ...

SOLUTION: $l = a - (n - 1)d$; $\therefore l = 64 - (14 - 1)3 = 25$, *ans.*

3. Find the 1st term of the series . . . . 68, 71, 74, having 19 terms.

SOLUTION: $l = a + (n - 1)d$; $\therefore 74 = a + (19 - 1)3$; $a = 20$, *ans.*

4. If the extremes of an arithmetical series are 3 and 15, and the number of terms 7, what is the common difference?

SOLUTION: $l = a + (n - 1)d$; $\therefore 15 = 3 + (7 - 1)d$, and $d = 2$, *ans.*

5. If the extremes are 2 and 23, and the common difference 3, what is the number of terms?

SOLUTION: $l = a + (n - 1)d$; $\therefore 23 = 2 + (n - 1)3$, and $n = 8$, *ans.*

6. If the extremes are 5 and 32, and the number of terms 12, what is the sum of the series?

SOLUTION: $s = \frac{n}{2}(a + l)$; $\therefore s = \frac{12}{2}(5 + 32) = 222$, *ans.*

Exercise XXII

1. Find the 9th term of the series 3, 5, 7, ...

2. Find the 1st term of the series . . . . 53, 56, 59, having 10 terms.

3. The extremes are 1 and 51, and the number of terms is 76. What is the common difference?

4. If a is 2, and l is 17, and n is 6, what is d?

5. If the extremes are 5 and 75, and the common difference 5, what is the number of terms?

6. How many strokes does a clock make in 12 hours?

7. 40 potatoes are 2 yd. apart and the first is 2 yd. from a basket. How far will a boy travel who gathers them and puts them into the basket one at a time?

8. Find s; given $a = 10$, $n = 6$, $d = -4$. Write out the series.

9. Find n; given $s = 36160$, $a = 40$, $l = 600$.

10. What is the sum of the first fifty odd numbers?

11. What is the sum of the first 50 numbers divisible by 7?

12. A body, if left unsupported, will fall by its own weight during the 1st second, 16.08 ft. (N. Y.); in each succeeding second, it will fall 32.16 ft. farther than in the preceding second; how far will it fall in the 11th second? In 11 seconds?

13. A boy throws a stone into the air, and it strikes the ground in 4 sec. How high did the stone go?

14. A bag of sand dropped from a balloon and fell to the earth in a quarter of a minute. How high was the balloon?

15. A body passed over 787.92 feet during its fall; what was the time required?

Remember that $d = 32.16$. Combine the two formulas for problems similar to the 15th; thus,

$$\text{Since } l = a + (n-1)d,\ s = \frac{n}{2}[a + a + (n-1)d].$$

Substituting in the 2d formula, we have

(1) $787.92 = \frac{n}{2}[16.08 + 16.08 + (n - 1)\,32.16]$

(2) $787.92 = \frac{n}{2}(32.16 + 32.16\,n - 32.16)$

(3) $787.92 = 16.08\,n^2$

(4) $49 = n^2$

(5) $7 = n$; $\therefore$ the time was 7 seconds.

16. A stone is thrown horizontally from the top of a tower 257.28 ft. high, with a velocity of 60 ft. a second. Where will it strike the ground?

17. A body falls freely for 6 seconds. What is the space traversed during the last 2 seconds of its fall?

18. A body falls from a certain height; 3 seconds after it has started, another body falls from the height of 787.92 feet; from what height must the first fall, if both are to reach the ground at the same instant?

II. GEOMETRICAL PROGRESSION

200. A **geometrical progression** is a series of numbers increasing or decreasing by a constant multiplier.

201. If the multiplier is greater than a unit, the series is **ascending.** If the multiplier is less than a unit, the series is **descending.**

202. The **ratio** is the constant multiplier.

203. In a geometrical progression there are *five parts,* any three of which being given, the other *two* can be found. The five parts with which we are concerned are:

1. The first term, represented by *a*.
2. The last term, " " *l*.

3. The number of terms, represented by n.
4. The ratio, " " r.
5. The sum of all the terms, " " s.

204. The **two fundamental formulas** of geometrical progression are:

I. The formula for the last term, $l = ar^{n-1}$.

This formula is derived from a series as follows:

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(n).
a,	ar,	ar^2,	ar^3,	ar^4,	ar^5,	ar^6, ...	l.

From the series we observe that *any* term consists of the first term multiplied by the ratio raised to the power indicated by a number which is 1 *less than the number of the term.* Hence, for the nth term, we have ar^{n-1}; or, letting l stand for the nth term, or last term, $l = ar^{n-1}$.

Study the formula and then *write the rule* for finding the *last term* in a geometrical series.

From the above formula, derive the formula for finding the *first term;* the *ratio. Write the rule* corresponding to each formula.

II. The formula for the sum of the series,

$$s = \frac{ar^n - a}{r - 1}.$$

This formula may be derived as follows:

$$\begin{aligned} s &= a + ar + ar^2 + ar^3 + \ldots\ ar^{n-2} + ar^{n-1}. \\ rs &= ar + ar^2 + ar^3 + \ldots\ ar^{n-2} + ar^{n-1} + ar^n. \end{aligned}$$

$$rs - s = ar^n - a.$$

$$s(r - 1) = ar^n - a.$$

$$s = \frac{ar^n - a}{r - 1}, \text{ or } s = \frac{a(r^n - 1)}{r - 1}, \text{ or } s = \frac{lr - a}{r - 1}.$$

Study this formula, and then *write the rule* for finding the sum of the geometrical series. From the formula above, derive the formula for finding the *first term;* the *last term;* the *ratio. Write the rule* corresponding to each formula.

EXAMPLES

1. Find the 8th term of the series 2, 4, 8, 16, ...

SOLUTION: $l = ar^{n-1}$; $\therefore$ $l = 2 \times 2^7 = 256$, *ans.*

2. Find the first term of the series . . . . 48, 96, 192, having 6 terms.

SOLUTION: $l = ar^{n-1}$; $\therefore$ $192 = a \times 2^5$; $a = 6$, *ans.*

3. Given $a = 6$, $l = 1536$, $r = 4$; find n.

SOLUTION: $l = ar^{n-1}$; $\therefore$ $1536 = 6 \times 4^{n-1}$.

$$256 = 4^{n-1}.$$

$$4^4 = 4^{n-1}; \therefore n - 1 = 4$$

$$n = 5, \textit{ ans.}$$

4. Given $a = 5$, $l = 5000$, $n = 4$; find r.

SOLUTION: $l = ar^{n-1}$; $\therefore$ $5000 = 5 \times r^3$; $r = \sqrt[3]{1000} = 10$, *ans.*

5. Given $a = 5$, $l = 6480$, $r = 6$; find s.

SOLUTION: $s = \frac{rl - a}{r - 1}$; $\therefore$ $s = \frac{6 \times 6480 - 5}{6 - 1} = 7775$, *ans.*

Exercise XXIII

1. Find the 11th term of the series 2, 6, 18, 54, ...
2. Find the 8th term of the series $3, 3^2, 3^3, 3^4$, ...
3. Find the 5th term of the series $1, \frac{1}{4}, \frac{1}{16}$, ...
4. Find the sum of $1, \frac{1}{2}, \frac{1}{4}$, etc. to infinity.

NOTE.—In a descending series with an *infinite* number of terms, the last term is 0, and the formula is written

$$s = \frac{-a}{r-1} = \frac{a}{1-r}.$$

5. Find the sum of $4, 2, 1, \frac{1}{2}$, etc. to infinity.
6. Insert 4 geometric means between 6 and 192.

7. Write a series of 4 terms, whose 1st term is 4 and whose ratio is $\frac{1}{2}$.

8. If a frog should jump 10 feet the first jump, 5 feet the second, 2.5 feet the third, etc. forever, how far would it go?

9. If a child should receive one cent at birth, 2 cents on the second birthday, 4 cents on the third, etc., how much would he be worth when 21 years of age?

NOTE.—Problems in compound interest may be solved by the principles of geometrical progression.

The terms are as follows:

a = principal.
r = 1 + rate per cent for one interval.
n = number of intervals + 1.
l = amount.

10. Find the amount of \$250 for 4 years, at 6% per annum compound interest.

SOLUTION: $l = ar^{n-1}$; $\therefore l = \$250 \times 1.06^4 = \$315.619$ +, *ans.*

11. Find the amount of \$400 for 3 years, at 6% per annum compound interest.

12. Find the compound interest of \$800 for 5 years, at 5%.

13. What principal will amount to \$322.51 in 24 years, at 5% per annum compound interest?

14. In how many years will \$80 amount to \$106.48, at 10% compound interest?

15. At what rate per cent compound interest will \$750 amount to \$946.86, in four years?

NOTE.—Circulating decimals may be changed to common fractions by using the principles of geometrical progression. Since a circulate always forms an infinite decreasing series, this formula is used:

$$s = \frac{a}{1-r}$$

16. Change $0.\dot{2}\dot{7}$ to an equivalent common fraction.

(1) $0.\dot{2}\dot{7} = 0.27 + 0.0027 + 0.000027 + \ldots$

(2) Given $a = 0.27$, $r = 0.01$, $n =$ infinity, to find s.

(3) $s = \frac{0.27}{1-0.01} = \frac{27}{99} = \frac{3}{11}$, *ans.*

17. Express as common fractions:

(1) $0.\dot{3}$

(2) $0.\dot{0}\dot{1}$

(3) $0.\dot{2}4\dot{7}$

(4) $0.152\dot{7}$

(5) $0.\dot{4}\dot{5}$

(6) $0.\dot{3}1\dot{8}$

CHAPTER VIII

METHOD OF ATTACK

205. Often pupils become discouraged and fail in arithmetic, because they neither understand the real nature of mathematical problems nor have any definite method of attacking them. While under each subject model solutions are given for problems arising under that special head, it is thought wise to give in this chapter some suggestions and examples which will be helpful in the solution of problems in general.

206. The Nature of Problems.—A problem consists of at least two parts: a *known part* and an *unknown part.*

1. The **known part** is a statement of conditions or relations forming a *basis* from which the unknown part may be determined.
2. The **unknown part** is frequently in the form of a *question* and is always the *part required.*

Problem: If 8 tops cost 40c., what will 17 tops cost?

PARTS:	1. Known:	8 tops cost 40c.
	2. Unknown:	What will 17 tops cost?

Sometimes the *known part* is not *expressed,* but *implied,* as in the following:

How many quarts in 10 gallons?

The known part is, there are 4 quarts in 1 gallon.

NOTE.—Before one attempts to solve a problem, he should determine the two parts and see clearly that a proper relation exists between them.

207. Classification.—Problems of one basis, considered with reference to the relation of the *given part* to the *required part,* are divided by Prof. J. A. Ferrell into *four classes,* as follows:

Class I. Given a number, to find (1) a part of it, or (2) a multiple of it; as,

1. What is the value of $\frac{3}{5}$ of an article which cost $7.20?

2. What will 50 books cost, at $2.50 each?

Class II. Given a part of a number, to find (1) the number, (2) a multiple of it, or (3) a part of it; as,

1. 20% of a sale is $540. What was the whole amount of the sale?

2. $\frac{5}{13}$ of a certain farm is 25 acres. How many acres in 3 such farms?

3. $\frac{3}{4}$ of a certain article is worth $.75. What is $\frac{4}{5}$ of it worth?

Class III. Given a multiple of a number, to find (1) the number, (2) a part of the number, or (3) another multiple of it; as,

1. If 35 hats sell for $105, find the price of 1 hat.

2. At the rate of $120 in 2 months, how much money can be earned in $\frac{3}{5}$ of a month?

3. 12 pounds Troy equal 144 ounces; how many ounces do 7 pounds Troy equal?

Class IV. Given two numbers, to find (1) what part one is of the other, or (2) what multiple one is of the other; as,

1. A commission merchant charges $96 for making a sale of $3200. What per cent does he charge?

2. John has $.50 and George has $5. George's money is how many times John's?

In the four classes there are *ten types* of problems; show that the examples given illustrate the types.

208. Method of Reasoning.—In arithmetic, as in all other subjects, we reason from the known to the related unknown; but there are other principles especially applicable in this subject. Since *unity* is the basis of all numbers, in analysis the following principles are fundamental:

1. *Reason from one to many.*
2. *Reason from many to one.*

209. These principles may now be illustrated by

EXAMPLES

1. If 1 top costs 8 cents, what will 5 tops cost?

SOLUTION:

1. The cost of 1 top = 8 cents.
2. The cost of 5 tops = 5 × 8 cents = 40 cents.

∴ if 1 top cost 8 cents, 5 tops will cost 40 cents.

In this we reason from one to many.

2. If 8 books cost $12, what will 1 book cost?

SOLUTION:

1. The cost of 8 books = $12.
2. The cost of 1 book = $\frac{1}{8}$ of $12 = $1.50.

∴ if 8 books cost $12, 1 book will cost $1.50.

In this we reason from many to one.

3. If 15 pencils cost 90 cents, what will 8 pencils cost?

Solution:

1. The cost of 15 pencils = 90 cents.
2. The cost of 1 pencil = $\frac{1}{15}$ of 90 cents = 6 cents.
3. The cost of 8 pencils = 8 × 6 cents = 48 cents.

∴ if 15 pencils cost 90 cents, 8 pencils will cost 48 cents.

In this the two principles are combined.

Exercise XXIV

Using analysis, solve the following:

1. Solve the ten problems under **207.**

2. A merchant, owning $\frac{7}{8}$ of a ship, sells $\frac{3}{5}$ of his share for $16800: at this rate what is the value of the whole ship?

3. A man pays $350 a year for house rent, which is $\frac{14}{65}$ of his income: what is his income?

4. A school enrolls 208 boys, and $\frac{7}{15}$ of the pupils are girls: how many pupils in the school?

5. If $\frac{2}{3}$ in. on a map corresponds to 7 mi. of a country, what distance on the map represents 20 mi.?

6. A certain sum of money gains $\frac{5}{8}$ of itself, the total amount then being $728; what is the sum gained?

7. A man agreed to work 20 days for $3 a day and his board, and to pay $1 a day for board when idle; at the end of the time he received $44. How many days was he idle?

8. If a box 7 ft. long, 5 ft. wide, and 4 ft. deep holds 112 bushels, how deep is another box which is 20 ft. long and 9 ft. wide, and holds 864 bushels?

Note.—See second solution of problem 1 under 188.

9. A and B could have done a work in 15 days, but after working together 6 days, B was left to finish it, which he did in 30 days: in what time could A have finished it, if B had left at the end of the 6 days?

10. A ship has water in it and water is running in through a leak at a uniform rate. If 60 sailors can bail out the water in 8 hours, and 90 sailors can do it in 5 hours, in what time can 50 sailors do the work?

SOLUTION:

If 1 sailor in 1 hr. can do 1 unit of work,
then 60 sailors " 8 " " " 480 units " "
and 90 " " 5 " " " 450 " " "

The amount of water running in during 8 – 5, or 3 hours, = 480 – 450, or 30 units of work. Therefore, the flow in 1 hour = 10 units of work, or the work of 10 sailors. During the 8 hours that the 60 sailors worked, 10 sailors were keeping out the flow, while the remaining 50 were emptying the ship. Hence, the amount of water in the ship when the work begins is represented by 50 × 8, or 400 units of work.

Of the 50 sailors, 10 will keep out the flow, and the number of hours required for the remaining 40 to empty the ship will be the number of times that 40 units of work is contained in 400 units of work; that is, 10 hours.

11. There is coal now on the dock, and coal is running on also from a shoot, at a uniform rate. Six men can clear the dock in one hour, but 11 men can clear it in 20 minutes: how long will it take 4 men?—R. N. H., p. 406.

12. If $\frac{7}{8}$ of a number is 35, how much is $\frac{3}{8}$ of it?

13. Three boys had 169 apples, which they shared in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. How many did each boy receive?

14. The 9th term of a geometric series is 137781, and the 13th term 11160261: what is the 4th term?

15. Express as common fractions: $.\dot{9}6\dot{3}$; $.\dot{3}7\dot{8}$; $.20\dot{4}\dot{5}$.

16. Change 52364_7 to the decimal scale.

17. Express in the quaternary scale, 5439.

18. A room is 21 ft. long, 16 ft. wide, and 12 ft. high. How far is it from an upper corner to the opposite lower corner?

19. At what time between 3 and 4 o'clock will the minute hand and the hour hand be together?

PRINCIPLES USED IN TIME PROBLEMS

1. The distance moved by the minute hand (min. h.) = 12 times the distance moved by the hour hand (hr. h.).
2. The distance gained by the min. h. = 11 times the distance moved by the hr. h. Therefore,
3. Every 12 spaces moved by the min. h. = 11 spaces gained.
4. 1 space gained by min. h. = $\frac{12}{11}$ spaces moved by it.

Solution:

1. Since 1 space gained = $\frac{12}{11}$ spaces moved,

2. ∴ 15 spaces gained = 15 × $\frac{12}{11}$ spaces moved = $16\frac{4}{11}$ spaces moved.

∴ the hands will be together at $16\frac{4}{11}$ min. past 3 o'clock.

20. At what time between 2 and 3 o'clock will the hands form a right angle?

Solution:

In this problem the min. h. must *gain* 25 spaces to form a right angle with the hr. h.

1. Since 1 space gained = $\frac{12}{11}$ spaces moved,

2. ∴ 25 spaces gained = 25 × $\frac{12}{11}$ spaces moved = $27\frac{3}{11}$ spaces moved.

∴ the hands will form a right angle at $27\frac{3}{11}$ min. past 2 o'clock.

21. At what time between 3 and 4 o'clock will the min. h. be opposite the hr. h.?

22. At what time between 3 and 4 o'clock is 3 midway between the two hands?

Suggestion.—The sum of the distances moved by the two hands = 15 spaces.

The min. h. dist. = $\frac{12}{13}$ of 15 spaces = $13\frac{11}{13}$ spaces.

23. At what time between 6 and 7 o'clock will 6 be midway between the two hands?

24. At what time between 5 and 6 o'clock are the two hands perpendicular to each other?

25. What time is it when $\frac{5}{8}$ of the time past noon equals $\frac{3}{4}$ of the time till midnight?

26. A cistern containing 480 gallons can be emptied by two pipes in 4 and 5 minutes, respectively. If both pipes are left open, in what time will they empty the cistern?

27. A reservoir has 3 pipes; the first can fill it in 10 days, the second in 16 days, and the third can empty it in 20 days. In what time will the reservoir be filled if they are all allowed to run at the same time?

28. The rate of the current of a river is 4 miles an hour. How far up the river can a boat go and return in 12 hours, if the boat's rate of travel in still water is 8 miles an hour?

Solution:

1. Boat's own rate of travel up stream = 8 mi. an hr.
2. Distance driven back by the current = 4 " " "
3. Therefore the rate of boat up stream = 4 " " "
4. Boat's own rate of travel down stream = 8 " " "
5. Distance carried by the current = 4 " " "
6. Therefore the rate of boat down stream = 12 " " "

7. The time to go 1 mi. up stream = $\frac{1}{4}$ hr.
8. The time to go 1 mi. down stream = $\frac{1}{12}$ hr.
9. The time to go 1 mi. up and return = $\frac{1}{4}$ hr. + $\frac{1}{12}$ hr. = $\frac{1}{3}$ hr
10. Since in $\frac{1}{3}$ hr. the boat can go and return 1 mi.,
11. In 1 " " " " " " " 3 mi.,
12. " 12 " " " " " " " 36 mi.

∴ in 12 hr. the boat can go 36 mi. and return.

29. The rate of the current of a river is 4 miles an hour. How far *down* the river can a boat go and return in 18 hours, if the boat's rate of travel in still water is 6 miles an hour?

30. In a river in which the rate of the current is 1.5 miles an hour, a boat travels 18 miles down stream in 3 hours. In what time can the boat return?

31. In a pasture where the grass is growing at a uniform rate, 13 cows can eat off the grass in 5 weeks, and 9 cows can eat off the grass in 9 weeks. How long will it take 7 cows to eat it?

32. A wolf is 84 leaps in advance of a dog. The wolf takes 8 leaps to the dog's 6, but 2 of the dog's leaps equal 5 of the wolf's. How many leaps must the dog take to catch the wolf?

SOLUTION:

1. The dog takes 6 leaps to the wolf's 8 leaps.
2. " " " 1 leap " " " $\frac{4}{3}$ "
3. But 2 of the dog's leaps = 5 wolf-leaps.
4. ∴ 1 " " " " = $\frac{5}{2}$ " "
5. $\frac{5}{2}$ wolf-leaps − $\frac{4}{3}$ wolf-leaps = $\frac{7}{6}$ wolf-leaps.
6. For the dog to gain $\frac{7}{6}$ wolf-leaps, he must take 1 leap.
7. " " " " " 1 wolf-leap, " " " $\frac{6}{7}$ "
8. " " " " " 84 wolf-leaps, " " " 72 leaps.

∴ the dog can catch the wolf in 72 leaps.

33. A fox is 60 leaps ahead of a hound, and takes 4 leaps while the hound takes 3; but 1 of the hound's equals 2 of the fox's leaps. How many leaps must the hound take to catch the fox?—Milne, p. 349.

34. A boat goes 18 miles an hour down stream, and 12 miles an hour up stream: if it is 20 hours longer in coming up than in going down, how far down did it go?

35. A boy is flying a kite with a string 225 feet long. If it is 112 feet from where the boy stands to a point directly under the kite, how high is the kite, allowing 13 feet for the sag of the string?

CHAPTER IX

DENOMINATE NUMBERS

210. A **denominate number** (Latin *denominare,* to name) is a number in which the unit or units are named.

211. A **simple denominate number** is one composed of but one denomination; as, 5 tons.

212. A **compound denominate number** is one composed of two or more denominations; as, 6 bu. 3 pk.; 5 yd. 2 ft.

213. **Reduction** is the process of changing a number from one denomination to another without changing its value. Reduction is of two kinds—*ascending* and *descending.*

214. **Reduction ascending** is the process of changing a number from one denomination to a higher denomination. Thus, 16 pk. = 4 bu.

215. **Reduction descending** is the process of changing a number from one denomination to a lower denomination. Thus, 4 bu. = 16 pk.

TABLES

216. The tables given here are those which the ordinary citizen is most likely to need. If one is to enter a business in which special measures are used, the tables should be learned then as a part of his preparation for that business.

I. MEASURES OF EXTENSION

LINEAR MEASURE

Common Measure

12 inches (in.) = 1 foot (ft.).
3 feet = 1 yard (yd.).
5½ yards, or 16½ feet = 1 rod (rd.).
320 rods, or 5280 feet = 1 mile (mi.).

Surveyors' Measure

7.92 inches = 1 link (li.).
25 links = 1 rod.
100 links = 1 chain (ch.).
80 chains = 1 mile.

SQUARE MEASURE

144 square inches (sq. in.) = 1 square foot (sq. ft.).
9 square feet = 1 square yard (sq. yd.).
30¼ square yards = 1 square rod (sq. rd.).
160 square rods = 1 acre (A.).
640 acres = 1 square mile (sq. mi.).
1 mile square = 1 section.
36 sections = 1 township.

NOTE.—In measuring roofs, etc., 100 sq. ft. = 1 square.

CUBIC MEASURE

1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.).
27 cubic feet = 1 cubic yard (cu. yd.).
128 cubic feet = 1 cord.

II. MEASURES OF CAPACITY

LIQUID MEASURE

4 gills (gi.) = 1 pint (pt.).
2 pints = 1 quart (qt.).
4 quarts = 1 gallon (gal.).
1 gallon = 231 cu. in.

DRY MEASURE

2 pints = 1 quart.
8 quarts = 1 peck (pk.).
4 pecks = 1 bushel (bu.).
1 bushel = 2150.42 cu. in.

III. MEASURES OF WEIGHT

AVOIRDUPOIS WEIGHT

16 ounces (oz.) = 1 pound (lb.).
100 pounds = 1 hundredweight (cwt.).
20 hundredweight = 1 ton (T.).

Note.—This table is used in weighing almost everything in common use, except gold and silver.

TROY WEIGHT

24 grains (gr.) = 1 pennyweight (pwt.).
20 pennyweights = 1 ounce.
12 ounces = 1 pound.

Note.—Troy weight is used for precious metals.

APOTHECARIES' WEIGHT

20 grains (gr.) = 1 scruple (sc., or ℈).
3 scruples = 1 dram (dr., or ʒ).
8 drams = 1 ounce (oz., or ℥).
12 ounces = 1 pound (lb.).

Note.—This table is used in selling drugs at retail.

COMPARATIVE TABLE OF WEIGHTS

437.5 Troy grains = 1 oz. Avoirdupois.
480 Troy grains = 1 oz. Troy = 1 oz. Apothecaries'.
1 oz. Avoirdupois = $\frac{175}{192}$ oz. Troy = $\frac{175}{192}$ oz. Apothecaries'.
7000 Troy grains = 1 lb. Avoirdupois.
5760 Troy grains = 1 lb. Troy = 1 lb. Apothecaries'.
1 lb. Apothecaries' = $\frac{144}{175}$ lb. Avoirdupois.

IV. CIRCULAR MEASURE

60 seconds (″) = 1 minute (′).
60 minutes = 1 degree (°).
30 degrees = 1 sign (S.).
360 degrees = 1 circumference of a circle.

NOTE.—This table is used in measuring angles, and in estimating latitude and longitude.

V. TIME MEASURE

60 seconds (sec.) = 1 minute (min.).
60 minutes = 1 hour (hr.).
24 hours = 1 day (da.).
7 days = 1 week (wk.).
365 days = 1 common year.
366 days = 1 leap year.
100 years = 1 century.

VI. MEASURES OF VALUES

U. S. MONEY

10 mills (m.) = 1 cent (ct.).
10 cents = 1 dime (d.).
10 dimes = 1 dollar ($).
10 dollars = 1 eagle (E.).

ENGLISH MONEY

4 farthings = 1 penny (1d.).
12 pence = 1 shilling (1s.).
20 shillings = 1 pound (£1).
£1 = $4.8665 U. S.

VII. MISCELLANEOUS MEASURES

PAPER MEASURE

24 sheets = 1 quire.
20 quires = 1 ream.
2 reams = 1 bundle.
5 bundles = 1 bale.

COUNTING

12 things = 1 dozen (doz.).
12 dozen = 1 gross.
12 gross = 1 great gross.
20 things = 1 score.

REDUCTION

217. The principles involved in reduction are the same in all the measures; therefore, one example in reduction ascending and one in reduction descending will be sufficient for models.

1. Reduce 684 pints to higher denominations.

SOLUTION:

$$\text{I.}\begin{cases}\text{1. Since 2 pints} = \text{1 quart.}\\ \text{2. } \therefore \text{ 1 pint} = \tfrac{1}{2}\text{ quart.}\\ \text{3. And 684 pints} = 684 \times \tfrac{1}{2}\text{ quart} = \text{342 quarts.}\end{cases}$$

$$\text{II.}\begin{cases}\text{1. Since 8 quarts} = \text{1 peck.}\\ \text{2. } \therefore \text{ 1 quart} = \tfrac{1}{8}\text{ peck.}\\ \text{3. And 342 quarts} = 342 \times \tfrac{1}{8}\text{ peck} = \text{42 pecks and 6 qt.}\end{cases}$$

III.
1. Since 4 pecks = 1 bushel.
2. ∴ 1 peck $= \frac{1}{4}$ bushel.
3. And 42 pecks $= 42 \times \frac{1}{4}$ bushel = 10 bu. and 2 pk.
∴ 684 pints = 10 bu. 2 pk. 6 qt.

2. Reduce 10 bu. 2 pk. 6 qt. to pt.

Solution:

I.
1. Since 1 bu. = 4 pk.
2. ∴ 10 bu. $= 10 \times 4$ pk. = 40 pk.
3. 40 pk. + 2 pk. = 42 pk.

II.
1. Since 1 pk. = 8 qt.
2. ∴ 42 pk. $= 42 \times 8$ qt. = 336 qt.
3. 336 qt. + 6 qt. = 342 qt.

III.
1. Since 1 qt. = 2 pt.
2. ∴ 342 qt. $= 342 \times 2$ pt. = 684 pt.
∴ 10 bu. 2 pk. 6 qt. = 684 pt.

Exercise XXV

1. Reduce 8 bu. 3 pk. 1 qt. 1 pt. to pints.
2. Reduce 5 mi. 17 rd. 2 yd. to feet.
3. Reduce 2 mi. 25 ch. 12 li. to links.
4. Reduce 2 lb. 6 oz. 12 pwt. 6 gr. to grains.
5. What part of 3 wk. is 4 da. 16 hr. 30 min.?
6. How many acres in four townships?
7. How many cubic inches in 5 gallons?
8. Reduce 40 sq. rd. 25 sq. yd. 7 sq. ft. to sq. ft.
9. How many cords of wood in 846 cu. ft. of wood?
10. Reduce 52875 cu. in. to higher denominations.
11. How many gills of water in 462 cu. in. of water?
12. How many bu. of wheat in 430084 cu. in. of wheat?

13. Reduce 1728 pt. to higher denominations.

14. Reduce 4 T. 15 cwt. 6 lb. 3 oz. to ounces.

15. Reduce 6 lb. 7℥ 5ʒ 2℈ 18 gr. to grains.

16. Reduce 5286 Troy gr. to higher denominations.

17. Reduce 66 feet to rods.

18. Troy weight is used in weighing gold, and Avoirdupois in weighing feathers. Which is heavier, a pound of gold or a pound of feathers? An ounce of gold or an ounce of feathers?

19. Reduce 5287″ to higher denominations.

20. Reduce 7 wk. 3 da. 6 hr. 8 min. 5 sec. to seconds.

21. How many seconds in Feb. of 1904?

22. Express £8 15s. 8d. in U. S. money.

23. How many sheets of paper in 4 bales?

24. How many dozen in 16 gross?

25. Reduce 146 lb. Troy weight to Avoirdupois weight.

26. Reduce .216 gr. to oz. Troy.

27. How many sq. ft. in the four sides of a room 21.5 ft. long, 16.5 ft. wide, and 13 ft. high?

28. Reduce 96 oz. Avoirdupois to oz. Troy.

29. Reduce 10240 sq. rd. to sq. ch.

30. Allowing 800 shingles to a square, how many shingles will be required to cover the roof of a house which is 60 feet long, and 15 feet from the comb to the eaves?

THE FOUR FUNDAMENTAL OPERATIONS

218. The processes of *addition, subtraction, multiplication,* and *division* of compound denominate numbers are based upon the same principles as those governing similar operations in simple numbers. In simple numbers, however, the scale is uniform, while in compound numbers the scales vary.

EXAMPLES

1. Add 2 bu. 1 pk. 1 pt. and 4 bu. 3 pk. 5 qt. 1 pt.

OPERATION.

	4	8	2	
bu.	pk.	qt.	pt.	
2	1	0	1	
4	3	5	1	
6	4	5	2	
7	0	6	0,	result.

SUGGESTION.—The reduction may be made as the additions are made, if desired; but one is less likely to make a mistake, if the additions are all made first and the reductions afterwards.

2. Subtract 4 yd. 2 ft. 6 in. from 8 yd. 1 ft. 10 in.

OPERATION.

	3	12	
yd.	ft.	in.	
8	1	10	
4	2	6	
3	2	4,	result.

EXPLANATION.—From 8 yd., 1 yd. is taken and reduced to ft., and added to the 1 ft., making 4 ft. Now, 4 ft. − 2 ft. = 2 ft. Only 7 yd. remained above. 7 yd. − 4 yd. = 3 yd.

3. Multiply 4 bu. 3 pk. 2 qt. by 12.

OPERATION.

	4	8	
bu.	pk.	qt.	
4	3	2	
		12	
48	36	24	
57	3	0,	result.

EXPLANATION.—The multiplications are made first, and the reductions afterward.

4. Divide 27 bu. 3 pk. 3 qt. by 6.

OPERATION.

		4	8	2	
	bu.	pk.	qt.	pt.	
6)	27	3	3	0	
	4	2	4	1,	result.

EXPLANATION.—(1) 27 ÷ 6 = 4, rem. 3. Write the 4 in the result, and reduce the 3 bu. to pk.

(2) 3 bu. + 3 pk. = 15 pk. 15 ÷ 6 = 2, rem. 3. Write the 2 in the result, and reduce 3 pk. to qt.

(3) 3 pk. + 3 qt. = 27 qt. 27 ÷ 6 = 4, rem. 3. Write the 4 in the result and reduce the 3 qt. to pt.

(4) 3 qt. = 6 pt. 6 ÷ 6 = 1. Write the 1 in the result. Result, 4 bu. 2 pk. 4 qt. 1 pt.

Exercise XXVI

1. Add:

(1)

hr.	min.	sec.
7	18	36
9	30	46
14	27	24

(2)

yd.	ft.	in.
45	1	11
28	2	7
19	0	·5

(3)

T.	cwt.	lb.
7	5	65
23	17	84
52	13	16

2. Subtract 15 bu. 1 pt. from 18 bu. 1 pk. 5 qt.

3. From 4 cwt. 38 lb. take 1 cwt. 49 lb.

4. From 6 mi. 85 ch. 1 rd. take 4 mi. 75 ch. 4 rd.

5. Multiply:
   (1) 24 bu. 5 qt. by 16.
   (2) 26 sq. yd. 6 sq. ft. 14 sq. in. by 9.
   (3) 6 lb. 3 oz. 17 pwt. 20 gr. by 7.
   (4) 14 yr. 9 mo. 7 da. by 14.

6. Divide:
   (1) 112 T. 16 cwt. 59 lb. by 7.
   (2) 65 gal. 2 qt. 1 pt. by 7.
   (3) 48 bu. 3 pk. 6 qt. by 9.
   (4) 72 yr. 9 mo. 20 da. by 12.

7. How many cups, holding one half pint each, can be filled from a coffee urn holding 2 gal. 3 qt. 1.5 pt.?

8. What will be the cost of 3 T. 6 cwt. 27 lb. of coal at $4.75 a ton?

9. What is the difference between a half of a foot square and a half of a square foot?

10. What is the difference between 6 dozen dozen and a half a dozen dozen?

11. Reduce 4 hr. 15 min. to the decimal of a day.

12. A man wishes to put 116 bu. 1 pk. 4 qt. of wheat into sacks that shall contain 2 bu. 1 pk. 4 qt. each. How many sacks will be required?

13. How many steel rails 30 feet long are needed to build 5 miles of railroad?

14. At $3.40 per square, what will be the cost of tinning both sides of a roof 40 feet in length, whose rafters are 20 ft. 6 in. long?

15. What part of an acre is a plot of ground 118 yd. long and 80 ft. wide?

16. How many minutes in the year 1904?

17. How many seconds in July?

18. In 10° 15′ 25″, how many seconds?

19. If a bale of cotton weighs 493 pounds, what will 6 bales cost at 10 cents a pound?

20. A parlor is 34 ft. long and 16 ft. 8 in. wide. What will it cost to cover the floor with Brussels carpet $\frac{3}{4}$ yd. wide, at $2.625 per yd.?

21. A farmer has three meadows. From the first, he raised 12 T. 17 cwt. 16 lb. hay; from the second, 14 T. 5 cwt. 4 lb.; and from the third, 7 T. 8 cwt. 13 lb. He reserved for his own use 12 T. 5 cwt. 8 lb., and sold the remainder at $18.00 per ton. How much did he receive?

22. A milkman adulterated 32 gallons of milk by adding a pint of water to every gallon of milk, and sold the entire quantity at the rate of 8 cents a quart. How much more

than the correct amount did the milkman dishonestly obtain?

VIII. LONGITUDE AND TIME

219. A **meridian** (Latin, *meridianus*, pertaining to mid-day) is an imaginary line extending due north and south from pole to pole.

220. A **prime meridian** is a meridian chosen as a *standard* from which to reckon longitude.

NOTE.—On October 1, 1884, a Prime Meridian Conference was held in Washington, and at this meeting the meridian extending through the Royal Observatory at Greenwich (a suburban city 5 miles southeast of London) was recommended to be the *prime meridian*. This recommendation has been adopted by nearly all the civilized nations. France still uses the meridian extending through the observatory at Paris.

221. **Longitude** (Latin *longus*, long) is the distance (expressed in circular measure) east or west from the prime meridian.

222. Since the earth makes one complete rotation upon its axis in 24 hours, every point on the earth's surface passes through 360° in that length of time. Therefore,

360° of longitude correspond to 24 hours of time.
15° “ “ “ “ 1 hour “ “
15′ “ “ “ “ 1 min. “ “
15″ “ “ “ “ 1 sec. “ “
1° “ “ corresponds “ 4 min. “ “
1′ “ “ “ “ 4 sec. “ “

Hence, the difference in longitude in degrees, minutes, and seconds is 15 times the difference in time in hours, minutes, and seconds, respectively.

223. *Longitude and time* gives rise to two cases:

1. To find the difference of longitude between two places when the difference of time is given.
2. To find the difference of time between two places when their longitudes are given.

EXAMPLES

1. The difference between the local times of two places is 4 hr. 3 min.: what is the difference of longitude between them?

SOLUTION:

1. Since 1 hr. of time corresponds to 15° of longitude,
2. ∴ 4 " " " correspond " 60° " "
3. Since 1 min. of time corresponds to 15′ of longitude,
4. ∴ 3 " " " correspond " 45′ " "

∴ the difference of longitude is 60° 45′.

From this solution we may obtain the

RULE.—**To find the difference of longitude between two places when the difference of time is given:**

Multiply the difference of time by 15, *and call the result degrees* (°), *minutes* (′), *and seconds* (″) *of longitude.*

2. The difference of longitude between two places is 28° 40′ 30″: what is the difference of time?

SOLUTION:

1. Since 1° longitude corresponds to 4 min. of time,
2. ∴ 28° " correspond " 112 " " "
3. Since 1′ " corresponds " 4 sec. " "
4. ∴ 40½′ " correspond " 162 " " "
5. But 112 min. + 162 sec. = 1 hr. 54 min. 42 sec.

∴ The difference of time is 1 hr. 54 min. 42 sec.

When the solution is understood, the pupil should be allowed to use the following

RULE.—**To find the difference of time between two places when the difference of longitude is given:**

Divide the difference of longitude by 15, *and call the result hours, minutes, and seconds of time.*

Exercise XXVII

1. The longitude of Denver is 105° 4′ W.; the longitude of Vicksburg is 90° 54′ W.; find the difference of sun-time.

2. The longitude of Berlin is 13° 23′ 45″ E.; the longitude of New Orleans is 90° W.; find the difference of sun-time.

QUESTION.—In problem 2, why do you *add* the numbers to get the *difference* of longitude?

3. The longitude of Austin is 97° 44′ W.; the longitude of St. Louis is 90° 14′ W.; find the difference of sun-time.

4. The longitude of San Francisco is 122° 24′ 30″ W.; the longitude of Honolulu, Hawaii, is 157° 52′ W.; find the difference of sun-time.

5. The longitude of Savannah, Ga., is 81° 5′ 26″ W., and the difference in sun-time between Savannah and El Paso, Tex., is 1 hr. 41 min. 30 sec. What is the longitude of El Paso?

6. If, at the next presidential election, the voting begins at sunrise, how much sooner will the polls open at Portland, Me., 70° 15′ 45″ W., than at Portland, Oregon, 122° 27′ 30″ W.?

7. A ship at sea finds that it is 3 hr. 34 min. p. m. by its

Greenwich chronometer when the sun is on the meridian; what is the ship's longitude?

8. The longitude of Honolulu is 157° 52' W., and the longitude of Sydney, Australia, is 151° 11' E.; when it is 6 o'clock a. m. on Saturday at Honolulu, what is the hour and day of the week at Sydney?

INTERNATIONAL DATE LINE

224. The **International Date Line** is the imaginary line, agreed upon by civilized nations, as the line which separates the lands in the Pacific Ocean where American date is kept from those where Asiatic date is kept. Navigators generally use the 180th meridian, but the *date line* varies slightly from this meridian. In passing through Bering Strait it bends to the east of the 180th meridian, and in avoiding the Aleutian Islands it reaches a point 10° west of the meridian. From a point about 8° to a point about 50° south of the equator it lies a little to the east of the 180th meridian. (The remainder of its course coincides with that meridian.)

225. The **day begins**, by agreement, at midnight on this date line. Therefore if it is Sunday forenoon when a steamer going *westward* reaches the line, it becomes Monday forenoon the moment the line is passed. But in going *eastward*, upon reaching the line on Sunday forenoon, it becomes Saturday forenoon the moment the line is passed. Thus the day that is dropped in going westward is "picked up" again on the return trip.

STANDARD TIME

226. Primarily for the convenience of railway traffic, a uniform system of time-keeping, known as **Standard Time,**

was agreed upon in 1883 by the principal railroad companies of the United States and Canada. By this system, *time meridians* 15° apart have been established. The United States uses the meridians 75°, 90°, 105°, and 120° west from Greenwich. Places within the belt extending 7½° east or west of a time meridian have the time of the time meridian. Therefore, the time between two places differs by *whole* hours, or not at all. The belts are named, beginning with the 75th meridian, as follows: Eastern Time, Central Time, Mountain Time, and Pacific Time.

227. Irregularities.—While the time meridians are exactly 15° apart, the belts using the several meridian times vary to suit the convenience of the railroads. This causes great irregularities. El Paso, Texas, has the same standard time as Savannah, yet there is a difference of 1 hr. 41 min. 30 sec. in local time. On the other hand, San Diego, California, has two hours later standard time than El Paso, although there is a difference of only 42 min. in local time.

Exercise XXVIII

1. The longitude of Buffalo is about 79° W. When it is noon in that city by standard time, what is the sun-time?

2. If a man travels from San Francisco to St. Louis, what change should he make in his watch?

3. Washington, D. C., is in longitude about 77° W. When it is 6 o'clock p. m. in that city by sun-time, what is the hour by a clock keeping standard time?

4. A man living in Galveston, Texas, observed that his clock, correct by sun-time, was 19 min. slower than the depot clock, correct by standard time. Find the longitude of Galveston.

5. Explain how the following information could reach the United States on Sunday, Feb. 28, 1904:

> For two hours yesterday morning [Monday, Feb. 29, 1904], the Japanese squadron bombarded Port Arthur, injuring the *Askold* and *Novik*, and again damaging the *Retvisan*. The Japanese fleet is reported to have escaped injury.—*Harper's Weekly.*

IX. MEASURES OF TEMPERATURE

228. Temperature is usually measured by a thermometer constructed upon one of the following scales:

(1) Fahrenheit (F.), boiling-point 212°, freezing-point 32°.
(2) Centigrade (C.), boiling-point 100°, freezing-point 0°.
(3) Réaumur (R.), boiling-point 80°, freezing-point 0°.

	F.	C.	R.
Boiling point of water	+ 212°	+ 100°	+ 80°
Freezing point of water	+ 32°	0°	0°
	+ 14°	− 10°	− 8°

TABLE OF EQUIVALENTS.

180° F. = 100° C. = 80° R.
1° " = $\frac{5}{9}$° " = $\frac{4}{9}$° "

NOTE.—Degrees above 0° on each scale are usually indicated by the sign +; below, by the sign −.

EXAMPLES, WITH SOLUTIONS

1. +86° F. corresponds to what readings on C. and R.?

(1) 86° F. − 32° F. = 54° F. = num. of degrees above freezing.
(2) 1° F. = $\frac{5}{9}$° C. = $\frac{4}{9}$° R.
(3) 54° F. = 54 × $\frac{5}{9}$ C. = 54° × $\frac{4}{9}$° R.
(4) 54° F. = 30° C. = 24° R.
(5) 54° F. + 32° F. = 86° F.
∴ 86° F. corresponds to 30° C. and 24° R.

2. −4° F. corresponds to what readings on C. and R.?

(1) − 4° F. − 32° F. = − 36° F. = num. degrees below freezing.
(2) 1° F. = $\frac{5}{9}$° C. = $\frac{4}{9}$° R.
(3) − 36° F. = − 36 × $\frac{5}{9}$° C. = − 36 × $\frac{4}{9}$° R.
(4) − 36° F. = − 20° C. = − 16° R.
(5) − 36° F. + 32° F. = − 4° F.
∴ − 4° F. corresponds to − 20° C. and − 16° R.

3. +24° R. corresponds to what readings on F. and C.?

(1) 80° R. = 180° F. = 100° C.
(2) 1° R. = $\frac{9}{4}$° F. = $\frac{5}{4}$° C.
(3) 24° R. = 24 × $\frac{9}{4}$° F. = 24 × $\frac{5}{4}$° C.
(4) ∴ 24° R. = 54° F. = 30° C.
(5) 54° F. + 32° F. = 86° F.
∴ + 24 R. corresponds to 86° F. and 30° C.

4. −10° C. corresponds to what readings on F. and R.?

(1) 100° C. = 180° F. = 80° R.
(2) 1° C. = $\frac{9}{5}$° F. = $\frac{4}{5}$° R.
(3) − 10° C. = − 10 × $\frac{9}{5}$° F. = − 10 × $\frac{4}{5}$° R.
(4) − 10° C. = − 18° F. = − 8° R.
(5) 32° F. − 18° F. = 14° F.
∴ − 10° C. corresponds to 14° F. and − 8° R.

Exercise XXIX

1. C. reads 10° above zero; what are the readings of F. and R.?

2. The temperature of the room is 68° F. Find the temperature in C.; in R.

3. Lead melts at 335° C. Find its melting point in F.

4. Silver melts at 1040° C. Find its melting point in R.; in F.

5. On a certain day the temperature fell from 95° F. to 10° C. How many degrees R. did it fall?

X. THE METRIC SYSTEM

229. The Metric System of weights and measures is used by all scientific writers throughout the civilized world. It is employed in the affairs of everyday life by most nations except the United States and Great Britain.

NOTE.—The Metric System was originated by the French. Two surveyors, Delambre and Méchain, measured an arc of the meridian from Dunkirk, France, to Barcelona, Spain (a distance of about 10 degrees). From this they computed the distance from the equator to the pole. The quadrant thus obtained was divided into ten million equal parts; one part was made the base of the system, and was called a *meter*. All measures are derived from the meter. The system was adopted by France in 1840, and legalized in the United States in 1868.

230. The **meter** (Greek *metron*, measure) is the unit of length, and is 39.37043 inches long.

The multiples and submultiples of the meter and of the units derived from it are based on the decimal scale.

The multiple units are designated by Greek prefixes. They are: deka (10), hekto (100), kilo (1000), and myria (10000). The submultiple units are designated by Latin prefixes. They are: deci (.1), centi (.01), and milli (.001).

An abbreviation of a *submultiple* begins with a *small letter*, while an abbreviation of a *multiple* begins with a *capital letter*.

Many of the denominations given in the tables are not in common use. Those usually employed in business or science are printed in bold-faced type.

MEASURES OF EXTENSION

231. The unit is the **meter**, about 39.37 inches.

LINEAR MEASURE

10 **millimeters** (mm) = 1 **centimeter** (cm) = .3937 in.
10 centimeters = 1 decimeter (dm) = 3.937 in.
10 decimeters = 1 **meter** (m) = 39.37 in.
10 meters = 1 dekameter (Dm) = 32.8 ft.
10 dekameters = 1 hektometer (Hm) = 19.9278 rd.
10 hektometers = 1 **kilometer** (Km) = .621+mi.
10 kilometers = 1 myriameter (Mm) = 6.21+ mi.

SQUARE MEASURE

100 sq. millimeters = 1 **sq. centimeter** (sq. cm.).
= .155+ sq. in.
100 sq. centimeters = 1 sq. decimeter (sq. dm.).
= 15.5+ sq. in.
100 sq. decimeters = 1 **sq. meter** (sq. m.).
= 1.196 + sq. yd.
100 sq. meters = 1 sq. dekameter (sq. Dm.).
= 119.6 + sq. yd.
100 sq. dekameters = 1 sq. hektometer (sq. Hm.)
= 2.47114 A.
100 sq. hektometers = 1 **sq. kilometer** (sq. Km.).
= 247.114 A = .386 (sq. mi.).

NOTE.—In measuring land, the square meter is called a *centare* (ca), the square dekameter an *are* (a), and the square hektometer a *hektare* (Ha).

CUBIC MEASURE

1000 cu. mm. = **1 cu. cm.**
1000 cu. cm. = 1 cu. dm.
1000 cu. dm. = **1 cu. m.**, or *stere*, = 1.308 cu. yd.

MEASURES OF CAPACITY

232. The unit in both liquid and dry measures is the **liter**, and it contains a *cubic decimeter*.

		Liquid	*Dry*
10 milliliters (ml)	= 1 **centiliter** (cl)	= .61 + cu. in.	
10 centiliters	= 1 deciliter (dl)	= .845 gi.	
10 deciliters	= 1 **liter** (l)	= 1.0567 qt.	= .908 qt.
10 liters	= 1 dekaliter (Dl)	= 2.64 + gal.	= 1.135 pk.
10 dekaliters	= 1 **hektoliter** (Hl)	= 26.4 + gal.	= 2.8375 bu.
10 hektoliters	= 1 kiloliter (Kl)	= 264 + gal.	

MEASURES OF WEIGHT

233. The unit is the **gram,** which is the weight of a *cubic centimeter* of distilled water at its greatest density (4° C).

		Avoirdupois
10 **milligrams** (mg)	= 1 **centigram** (cg)	= .15432 + gr.
10 centigrams	= 1 decigram (dg)	= 1.5432 + gr.
10 decigrams	= 1 **gram** (gr)	= 15.432 + gr.
10 grams	= 1 dekagram (Dg)	= .35273 + oz.
10 dekagrams	= 1 hektogram (Hg)	= 3.5273 + oz.
10 hektograms	= 1 **kilogram** (Kg)	= 2.20462 + lb.
10 kilograms	= 1 myriagram (Mg)	= 22.0462 + lb.
10 myriagrams	= 1 quintal (Q)	= 220.462 + lb.
10 quintals	= 1 **tonneau** (T)	= 2204.62 + lb.

REDUCTION

234. Reduction descending and ascending may be performed in the Metric System as in the other systems; but, since the scale is decimal, the reduction may be made by simply moving the decimal point.

EXAMPLES

1. Reduce 40000 dm. to Km.

PROCESS: 40000 dm. = 4.0000 Km. = 4 Km., result.

2. Reduce 6 Hm. 4 Dm. 2 m. to m.

PROCESS: 6 Hm. 4 Dm. 2 m. = 642 m., result.

3. Reduce 48365 dm. to higher units.

PROCESS: 48365 dm. = 4 Km. 8 Hm. 3 Dm. 6 m. 5 dm., result.

NOTE.—To divide by 10 repeatedly will give successively the remainders 5, 6, 3, etc., and the result will be as given above.

Exercise XXX

1. When was the Metric System adopted by France? How many years elapsed before it was legalized in the United States? How extensively is it used (a) in business, (b) in science? What advantage has it over the older systems?

2. How was the length of the meter fixed? How was the liter fixed? How was the gram fixed?

3. Define *milli, centi, deci; deka, hekto, kilo, myria.*

Reduce:

4. 4735 cm. to higher units.

5. 7 Mm. 8 Km. 6 Hm. 5 Dm. 4 m. to mm.

6. 236220 ft. to Dm.

7. 4.003 Dm. to mm.

8. 15 sq. Mm. to sq. m.

9. 540000 sq. cm. to sq. m.

10. 75 cu. Km. to cu. m.

11. 8 Hl. 4 l. to dl.

12. 75 bu. to Dl.

13. 7 Hl. to gal.

14. 6 Dg. to mg.

15. 50 Mg. to lb. av.

16. 5863 cg. to higher units.

17. Find the weight in grams of a cu. dm. of iron (sp. gr. 7.21).

NOTE.—The *specific gravity* (sp. gr.) of a substance is the ratio between its weight and the weight of an equal volume of water.

18. What is the weight in pounds of a cu. m. of ice (sp. gr. 0.92)?

19. The Eiffel tower at Paris is 300 m. high. How many feet higher is it than the Washington monument (555 ft. high)?

20. Find the cost of excavating 6.25 cu. m. of earth, at $0.0015 per cu. dm.

21. The nickel 5-cent piece is 20 mm. in diameter. How many laid side by side will reach 2020 Dm.?

22. A nickel 5-cent piece weighs 5 g.; a silver half-dollar weighs 12.5 g.; what is the combined weight of $12 in nickels and $40 in silver half-dollars?

23. If I buy 500 bu. of wheat at 75¢ a bushel, and sell it at $3 per Hl., do I gain or lose?

24. A merchant bought cloth at $1.15 per meter and sold it by the yard at a profit of 20%. How much did he get for it per yard?

25. A physician put up 500 dg. of quinine into pills of 8 cg. each; how many pills were there?

26. Find the capacity in Hl. of a barn 5 m. long, 4 m. wide, and 3 m. deep.

27. What per cent. of 185 Dg. is 740 dg.?

28. A room is 5.2 m. long, 4.5 m. wide, and 3.2 m. high. What will be the cost of plastering it at 35 cents per sq. m.?

XI. SPANISH LAND MEASURES

235. The old Spanish land measures are still used in Texas, and to some extent in New Mexico and in the southern part of California. These places were originally settled by the Spanish. The most important terms used are these: **vara, labor,** and **league.**

236. The **vara** (vr.), corresponding nearly to our yard, is the unit in linear measure. The vara was divided into three equal parts called *piés* (feet), and the *pié* was divided into twelve equal parts called *pulgadas* (inches). The *pié* and the *pulgada* are not much used in Texas.

LINEAR MEASURE

1 vara = $33\frac{1}{3}$ inches.

36 varas = 100 feet.

1900.8 varas = 1 mile = 5280 feet.

5000 varas = 1 league = 2.63+ miles.

SQUARE MEASURE

237. The **square vara** (sq. vr.) is the unit.

25000000 sq. vr. = 1 sq. league = 4428.4 acres.
1000000 sq. vr. = 1 labor = 177.136 acres.
26000000 sq. vr. = 1 league and labor = 4605.536 acres.
5645.376 sq. vr. = 1 acre.
3613040.64 sq. vr. = 1 section = 640 acres.

NOTE.—The term **league** usually means 4428.4 acres of land without regard to shape, or in Spanish, a *sitio de ganado mayor* (a site for large cattle). There is also a *sitio de ganado menor* (a site for small cattle), which contains 11111111 + sq. vr., or 1968.1777 acres. The expression "large cattle" has reference to *horses*, *cows*, *etc.*, and "small cattle" has reference to *sheep*, *goats*, *etc.* "Cattle" is used in the sense of "live stock."

The term **labor** is now applied to 1000000 sq. vr., or 177.136 acres of land, but the original idea was that the labor was a field for farming purposes attached to the league, which was used for pasture land.

The **porcion** (share) has no fixed size. Along the Rio Grande in Texas, a porcion usually contains 30000000 sq. vr., or 5314.08 acres.

Exercise XXXI

1. What is a vara? A labor? A league?

2. How many varas in 100 yards? In 320 rods?

3. How many varas in 8 miles?

4. Reduce 11404.8 varas to miles.

5. The distance from Austin to San Antonio is 152064 varas; find the distance in miles.

6. The distance from Fort Worth to Dallas is 30 miles; find the distance in linear leagues.

7. How many square yards in a labor?

8. How many varas in one side of a square league?

9. How many miles of fence does it require to inclose a square league of land?

10. How many acres in a rectangular field 684 varas in length and 345 varas in width?

11. Find the number of square varas in a ranch containing 30000 acres.

13. Find the number of sq. varas in a park 4876 feet in length and 3764 feet in width.

13. Find the number of square feet in a lot 98 varas in length and 56 varas in width.

14. Find the number of square leagues of land in a county 30 miles square.

15. Verify the following rules:

(1) **To reduce varas to feet,**—*multiply by 100, then divide by 36.*

(2) **To reduce feet to varas,**—*multiply by 36, then divide by 100.*

(3) **To reduce sq. vr. to acres**—*multiply by 177.136, then divide by 1000000.*

(4) **To reduce acres to sq. vr.,**—*multiply by 1000000, then divide by 177.136.*

CHAPTER X

MENSURATION

238. Mensuration (Latin *mensura,* a measure) is that branch of mathematics which treats of geometrical magnitudes. It is the application of arithmetic to geometry.

239. The **four fundamental geometric concepts** are the *point,* the *line,* the *surface,* and the *solid.* All but the point have magnitude.

1. A **point** is that which has position, but no magnitude.
2. A **line** is that which has only length. If a point moves, it generates a line.
3. A **surface** is that which has only *length* and *width.* If a line moves (not along itself), it generates a surface.
4. A **solid** is that which has *length, width,* and *thickness.* If a surface moves (not along itself), it generates a solid.

I. PLANE FIGURES

240. A **plane** is a surface such that any two points in it can be joined by a straight line which lies wholly in the surface.

241. A **plane figure** is any portion of a plane bounded by lines.

242. A **polygon** is a plane figure bounded by straight lines. The **perimeter** of a polygon is the sum of the lines bounding it.

243. The least number of straight lines which can inclose a plane is *three.*

A polygon having three sides is a *triangle;* four sides, a *quadrilateral;* five sides, a *pentagon;* six sides a *hexagon,* etc.

244. A **regular polygon** is one whose sides and whose angles are equal.

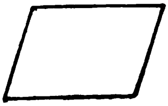
245. The **diagonal** of a polygon is the straight line joining two angles not adjacent.

246. The **base** is the side upon which a figure is supposed to stand.

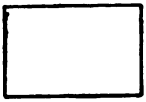
247. The **altitude** of a polygon is the perpendicular distance from the highest point to the line of the base.

248. The **center** of a regular polygon is the point within the polygon, equally distant from the middle points of the sides; and the **apothem** of such a polygon is the perpendicular line drawn from the center to the middle of a side.

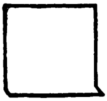
249. A **parallelogram** is a quadrilateral whose opposite sides are parallel.



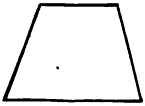
250. A **rectangle** is a parallelogram whose angles are right angles.



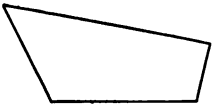
251. A **square** is a rectangle whose four sides are equal.



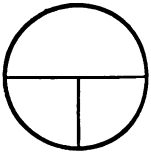
252. A **trapezoid** is a quadrilateral with only two sides parallel.



253. A **trapezium** is a quadrilateral with no two sides parallel.



254. A **circle** is a plane figure bounded by a curved line, every point of which is equally distant from the center. The boundary line of a circle is called the **circumference.** The **diameter** is a straight line drawn through the center of the circle, its end points being in the circumference. The **radius** is a straight line drawn from the center to the circumference.



255. The **area** of a plane figure is its amount of surface. Area is denoted by the number of square units a figure contains.

256. The **unit of surface** is a square whose edge is a *linear unit.*

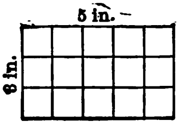
257. All **surface measurements** are based on the rectangle.

258. The **fundamental principle** is:

The area of a rectangle is (abstractly) equal to the product of the base by the altitude.

ILLUSTRATION

Find the area of a rectangle 5 in. long and 3 in. wide.



SOLUTION :

1. A surface 1 in. long and 1 in. wide = 1 sq. in.
2. “ “ 5 “ “ “ 1 “ “ = 5 × 1 sq. in. = 5 sq. in.
3. “ “ 5 “ “ “ 3 “ “ = 3 × 5 “ “ = 15 “ “

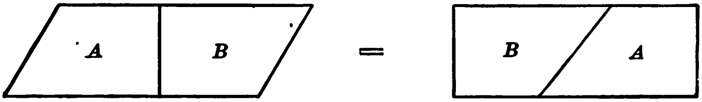
The solution suggests the following

RULE.—**To find the area of a rectangle:**

Multiply the base by the altitude.

NOTE.—For the sake of brevity, all rules of mensuration are stated as though the numbers were abstract.

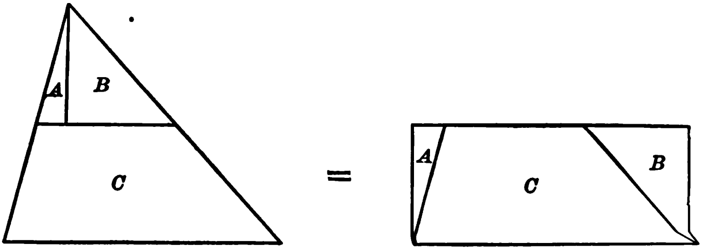
259. The **rule** for finding the area of a **parallelogram** is the same as the rule for finding the area of a **rectangle;** because the parallelogram can be converted into a rectangle having the *same base* and the *same altitude.* Thus,



260. RULE.—**To find the area of a triangle:**

Take half the product of the base by the altitude.

Reason: A triangle can be converted into a rectangle with the *same base and half the altitude.* Thus,



261. RULE.—**To find the area of a triangle when the altitude is unknown, but the three sides are given:**

Take the square root of the continued product of the half sum of its sides and the remainders found by subtracting each side from the half sum separately.

EXAMPLE

Find the area of a triangle whose sides are 6 in., 8 in., and 10 in.

SOLUTION:

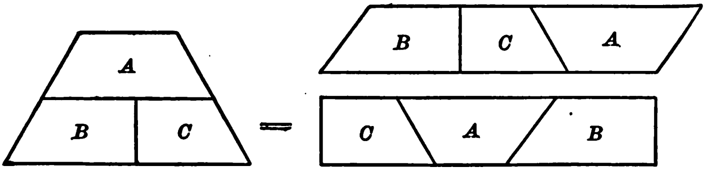
Let $s = \frac{1}{2}$ the sum of the sides,
And $a, b, c,$ = the sides, respectively.

RELATION: $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}.$
$\text{Area} = \sqrt{12 \times 6 \times 4 \times 2} = 24.$
$\therefore$ Area = 24 sq. in.

262. RULE.—**To find the area of a trapezoid:**

Take half the product of the sum of the bases by the altitude.

Reason: The trapezoid can be converted into a rectangle with a base equal to the *sum of the bases* of the trapezoid, and with an altitude equal to *half the altitude* of the trapezoid. Thus,



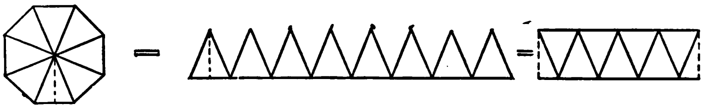
263. RULE.—**To find the area of a trapezium and other irregular polygons:**

Divide the trapezium or other irregular polygon into triangles and find the sum of the areas of the triangles.

264. Rule.—**To find the area of a regular polygon:**

Take half the product of the perimeter by the apothem.

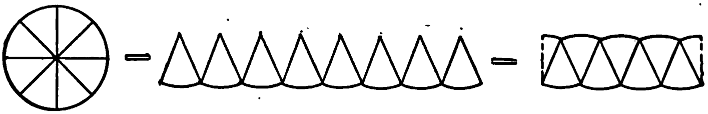
Reason: A regular polygon can be converted into a rectangle whose base is *half the perimeter* of the polygon, and whose *altitude* is the *apothem.* Thus,



265. Rule.—**To find the area of a circle:**

Take half the product of the circumference by the radius.

Reason:—A circle can be converted into an approximate rectangle whose base is half the circumference of the circle, and whose altitude is the *radius.* Thus,



266. The **ratio** between the circumference and diameter of a circle is 3.14159265 . . . ., which is usually represented by the Greek letter π (pi). For practical purposes, the value of π is considered 3.1416 or else $3\frac{1}{7}$. In this book, 3.1416 is used.

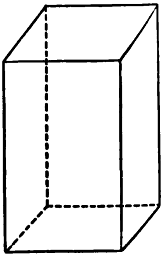
If c = circumference, d = diameter, and r = radius, the area of a circle $= \frac{c \times r}{2}$. Remembering that $c = \pi d$; $c = 2\pi r$; $d = \frac{c}{\pi}$; $d = 2r$; $r = \frac{d}{2}$, and $r = \frac{c}{2\pi}$, we may by substitution derive a number of formulas for the area of a circle.

Thus, area $= \frac{\pi dr}{2}$; area $= \pi r^2$; area $= \frac{1}{4}cd$; area $= \frac{1}{4}\pi d^2$; area $= \frac{c^2}{4\pi}$.

NOTE.—Instead of dividing by π, we may multiply by its reciprocal, $\frac{1}{\pi} = 0.3183 \ldots$.

II. SOLIDS

267. A **prism** is a solid whose ends, or bases, are equal and parallel polygons, and whose sides are parallelograms.



The **convex,** or **lateral, surface** of a prism is the sum of the surfaces of the sides. The entire surface is the sum of the convex surface and the areas of the bases.

268. RULE.—**To find the convex surface of a prism:**

Multiply the perimeter by the altitude.

Reason: The convex surface of a prism can be converted into a rectangle with the *same altitude* as the prism, and the *perimeter of the prism* for its base.

RULE.—**To find the entire surface of a prism :**

Add the areas of the bases to the convex surface.

269. The **solid contents** of a body is denoted by the number of cubic units it contains. The **cubic unit** is a cube whose edge is the *linear unit.*

270. The **fundamental principle** in cubic measurements is: *The volume, or solid contents, of a rectangular solid is equal to the continued product of its three dimensions.*

ILLUSTRATION

Find the solid contents of a rectangular box 6 in. long, 4 in. wide, and 3 in. high.

SOLUTION:

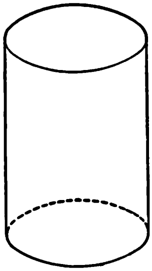
1. A solid 1 in. by 1 in. by 1 in. = 1 cu. in.
2. " " 6 " " 1 " " 1 " = 6 × 1 cu. in. = 6 cu. in.
3. " " 6 " " 4 " " 1 " = 4 × 6 " " = 24 " "
4. " " 6 " " 4 " " 3 " = 3 × 24 " " = 72 " "

The solution suggests the following

RULE.—**To find the solid contents of a rectangular solid:** *Multiply the length by the width, and this product by the height.*

271. RULE.—**To find the solid contents of a prism:** *Multiply the area of the base by the altitude.*

272. **A cylinder** is a solid whose bases are equal and parallel circles, and whose diameter is uniform throughout.



273. RULE.—**To find the convex surface of a cylinder:**

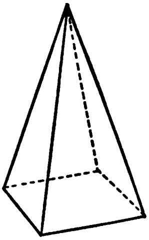
Multiply the circumference of the base by the altitude.

Reason: The convex surface of a cylinder can be converted into a rectangle with the *same altitude,* and with the *circumference of the base of the cylinder for its base.*

274. RULE.—**To find the solid contents of a cylinder:** *Multiply half the circumference by the radius, and the product by the altitude.*

Reason: A cylinder can be converted into an approximate rectangular solid with *half the circumference of the cylinder* for its base, with the *radius* for its *altitude,* and with the *altitude of the cylinder* for its *length.*

275. A **pyramid** is a solid whose base is a polygon, and whose sides are triangles meeting in a point called the **vertex.**



The **altitude** is the shortest distance from the vertex (called apex) to the plane of the base; the **slant height** is the shortest distance from the vertex to the middle point of one of the sides of the base.

276. Rule.—**To find the convex surface of a regular pyramid:**

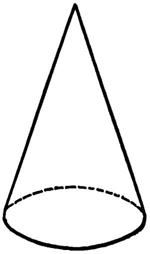
Multiply the perimeter of the base by half the slant height.

Reason: The convex surface of a pyramid can be converted into a rectangle with the *perimeter of the base* for its base, and *half the slant height* for its altitude.

277. Rule.—**To find the solid contents of a pyramid:**

Multiply the area of the base by one third of the altitude.

Reason: It is proved in geometry that the solid contents of a pyramid is equal to *one-third* of the solid contents of a prism with an *equal base* and the *same altitude.*



278. A **cone** is a solid whose base is a circle, and whose surface tapers uniformly to a point called the **vertex.**

279. Rule.—**To find the convex surface of a right cone:**

Multiply the circumference of the base by half the slant height.

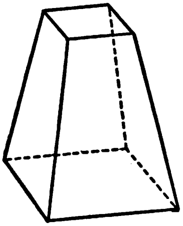
Reason: The convex surface of a cone is a sector of a circle, and can be converted into an approximate rectangle, with the *circumference of the base* for its base, and *half the slant height* for its altitude.

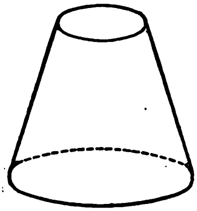
280. Rule.—**To find the solid contents of a cone:**

Multiply the area of the base by one-third of the altitude.

Reason: It is proved in geometry that the volume of a cone is equal to one-third of the volume of a cylinder with an *equal base* and the *same altitude.*

281. The **frustum** of a pyramid or cone is a solid which remains when a portion having the vertex is cut off by a plane parallel to the base.





282. Rule.—**To find the convex surface of a frustum of a pyramid or cone:**

Multiply half the sum of the perimeters of the bases by the slant height.

283. Rule.—**To find the solid contents of a frustum of a pyramid or cone:**

To the sum of the two bases add the square root of their product, and multiply the amount by one-third of the altitude.

284. A **sphere** is a solid bounded by a curved surface, every point of which is equally distant from a point within called the center.

285. RULE.—**To find the surface of a sphere:**

Multiply the circumference by the diameter.

NOTE.—The surface of a sphere is 4 times as great as the surface of a circle whose diameter equals the diameter of the sphere. Since the surface of a circle equals πr^2, the surface of a sphere may be represented by either $4\pi r^2$, or πd^2.

286. RULE.—**To find the solid contents of a sphere:**

Multiply the area of the surface by one-third of the radius.

Reason: A sphere may be regarded as composed of pyramids whose bases form the surface of the sphere, and whose common altitude is the radius of the sphere.

NOTE.—Since the surface of a sphere may be represented by $4\pi r^2$, the solid contents may be represented by

$$\frac{r}{3} \times 4\pi r^2 = \frac{4}{3}\pi r^3 = \frac{\pi d^2 r}{3} = \frac{\pi d^3}{6} = .5236d^3.$$

III. SIMILAR FIGURES

287. Similar figures are those that have the *same shape.* The parts similarly placed are said to be **homologous.**

288. Fundamental principles of similar figures:

1. Homologous lines are proportional.
2. Two similar surfaces are to each other as the squares of any two homologous lines.

3. Two similar solids are to each other as the cubes of any two homologous lines.

Exercise XXXII

1. Find the area of a rectangle 9 ft. 3 in. long and 3 ft. 9 in. wide.

2. Find the area of a parallelogram whose base is 42 rd. and altitude 17 rd.

3. How many tiles 6 in. square in a floor 36 ft. long and 20 ft. wide?

4. Find the area of a triangular field whose base is 72 rd. and altitude 16 rd.

5. Find the area of a triangular garden whose base is 7 ch. and whose altitude is 5 ch.

6. How many bricks 8 in. long and 4 in. wide will it take to pave a cellar floor 36 ft. long and 24 ft. wide?

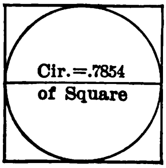
7. Find the area of a walk 12 ft. wide at one end, 6 ft. wide at the other end, and 120 ft. long.

8. Find the area of a circle whose radius is 6 ft.; whose diameter is 8 ft.

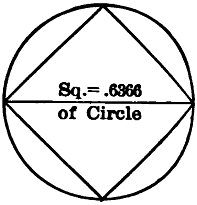
9. How long a rope will it take to fasten a horse to a post so that he may graze over 2 acres?

10. Show that the area of a circle $= \frac{\pi d^2}{4}$.

11. Show that the area of an inscribed circle is equal to .7854 of the area of the square.



12. Show that the area of an inscribed square is equal to .6366 of the area of the circle.



13. Show that the area of a square inscribed in a circle is equal to half the area of the square circumscribed about the same circle.

14. Show that the area of a circle inscribed in a square is equal to half the area of the circle circumscribed about the same square.

15. In a square field containing 100 acres, a circle is inscribed; how many acres in one corner cut off by the circle?

16. What is the thickness of the largest square stick of timber that can be cut out of a cylindrical log 17 inches in diameter?

17. Around a circular grass plot containing $3490\frac{2}{3}$ sq. yd. is a concrete walk 10 ft. wide. Find the number of sq. yd. in the walk.

18. Find the cost, at 1 cent a sq. ft., of painting a church spire whose base is 10 ft. sq. and whose slant height is 50 ft.

19. What is the convex surface of a cone whose base is 10 ft. in diameter, and whose slant height is 20 ft.?

20. What is the convex surface of a vat whose top is 8 ft. sq., whose base is 9 ft. sq., and whose slant height is 10 ft.?

21. Find the number of sq. ft. in a triangular lot whose sides are 40, 50, and 60 yd.

22. How many sq. ft. in a plank 16 ft. long, 27 in. wide at one end, and 21 in. wide at the other?

23. How many sq. ft. of pasteboard will be required to make a closed box 24 in. long, 16 in. wide, and 12 in. deep?

24. The diameter of a sphere is 48 in. Find its surface and volume.

25. The circumference of a circle equals the perimeter of a square, 100 ft. Which has the larger area?

26. Two sides of a rectangle are 30 ft. and 40 ft. Find the area of the circumscribed circle.

27. A horse is tied to the corner of a house 60 ft. long and 40 ft. wide by a rope 90 ft. long. Over how much ground can the horse go?

28. Find the difference between the volumes of a square prism circumscribed about, and one inscribed in, a cylinder the diameter of whose base is 3 ft. and whose altitude is 4 ft.

29. Show that a cube whose edge is 6 in. contains as many cu. in. as there are sq. in. in its surface.

30. Find the difference between the volumes of a 10-inch cube and the largest sphere that can be cut out of it.

31. A cylindrical cup 10 in. in diameter and 10 in. high is full of water. An iron ball 10 in. in diameter is put into the cup and some of the water runs over. How much water can remain in the cup?

32. Find the solid contents of the frustum of a cone whose upper base is 5 ft. in diameter, whose lower base is 8 ft. in diameter, and whose altitude is 7 ft.

33. What is the length of one edge of the largest cube that can be cut out of a sphere 9 in. in diameter?

34. The convex surfaces of two cones are to each other as 9 : 16; the altitude of the smaller cone is 21 in.; find the altitude of the other cone.

35. A sphere 5 in. in diameter weighs 15 pounds; what does a similar sphere weigh whose diameter is 10 in.?

36. A and B bought a ball of twine 6 in. in diameter for 24 cents. A wound off the twine until the ball was but 3 in. in diameter, and then gave the ball to B. How much should each pay?

37. Show that the area of a sphere whose diameter is 6 in. contains the same number of square inches as there are cubic inches in its volume.

SUGGESTION.—The volume of a sphere $= \frac{\pi d^3}{6}$.

38. Show that the diameter of a sphere multiplied by .57735 equals the edge of the largest cube that can be cut out of the sphere.

SUGGESTION.—The diameter of the sphere equals the diagonal of the cube. Therefore $d^2 = 3x^2$, letting x represent the edge. Let $d = 1$, and solve for x.

39. The area of a given rectangle 9 ft. wide is 180 sq. ft.; what is the area of a similar rectangle 24 ft. wide?

40. If a man 6 ft. tall weighs 162 pounds, what is the weight of a boy of similar build who is but 4 ft. tall?

CHAPTER XI

PERCENTAGE AND ITS APPLICATIONS—WITHOUT TIME

289. **Percentage** is a term applied to operations in which 100 is used as the basis of comparison. The term is also used to denote the result obtained by taking a given per cent of a number.

In percentage, the principles are the same as those used in operating with common and decimal fractions. There is no mathematical reason for separating percentage from decimal fractions, but the commercial world has so generally adopted 100 as the basis of comparison that it has become customary to treat percentage as a special subject.

290. **Per cent** is a shortened form of the Latin *per centum,* which means *by the hundred.*

291. The **sign** % is read *per cent;* thus, 5% is read 5 *per cent,* and may be expressed $\frac{5}{100}$ or, .05.

292. The **terms** used in percentage are *base, rate, rate per cent,* and *percentage.*

293. The **base** is the number on which the percentage is estimated.

294. The **rate** is the ratio (expressed in hundredths) of the percentage to the base.

295. The **rate per cent** is the *number* of hundredths in the rate. Thus, if the *rate* is $\frac{5}{100}$, the *rate per cent* is 5.

296. The **percentage** is the result obtained by taking a given per cent of a number.

297. Fundamental principle in percentage: The value of 100% of a quantity is the quantity itself.

Since the sign % means hundredths, $100\% = \frac{100}{100} = 1$. But $\frac{100}{100}$ of a quantity = the entire quantity.

This principle is used in the following

ILLUSTRATIVE EXAMPLES

1. Find 45% of \$650.

SOLUTION:

1. $\text{Since } 100\% \text{ of } \$650 = \$650,$
2. $\therefore \quad 1\% \text{ of } \$650 = \frac{1}{100} \text{ of } \$650 = \$6.50,$
3. $\text{and} \quad 45\% \text{ of } \$650 = 45 \times \$6.50 = \$292.50.$

∴ ———

SHORT PROCESSES:

$45\% \text{ of } \$650 = \frac{45}{100} \text{ of } \$650 = \$292.50.$ Or,

$$\begin{array}{rl} \$6.50 = & 1\%. \\ 45 & \\ \hline \$292.50 = & 45\%. \end{array}$$

2. \$72 is how many % of \$480?

SOLUTION:

1. $\text{Since } \$480 = 100\% \text{ of } \$480,$
2. $\therefore \quad \$1 = \frac{1}{480} \text{ of } 100\% \text{ of } \$480 = \frac{10}{48}\% \text{ of } \$480,$
3. $\text{and} \quad \$72 = 72 \times \frac{10}{48}\% \text{ of } \$480 = 15\% \text{ of } \$480.$

∴ ———

SHORT PROCESS:

$\$72 \div \$480 = .15 = 15\%,$ *ans.*

3. 40 is 8% of what number?

SOLUTION:

1. $\text{Since } 8\% \text{ of req. number} = 40,$
2. $\therefore \quad 1\%$ " " " $= \frac{1}{8} \text{ of } 40 = 5,$
3. and 100% " " " $= 100 \times 5 = 500.$

∴ ———

SHORT PROCESS:

If $\frac{8}{100} = 40$, $\frac{100}{100} = \frac{100}{8}$ of $40 = 500$, *ans.*

4. $300 is 20% more than what sum?

SOLUTION:

1. Since 120% of req. sum = $300,
2. ∴ 1% of req. sum = $\frac{1}{120}$ of $300 = $2.50,
3. and 100% of req. sum = 100 × $2.50 = $250.

∴ ———

SHORT PROCESS:

$300 ÷ $\frac{120}{100}$ = $250. ∴ $250 is the req. sum.

5. $400 is 20% less than what sum?

SOLUTION:

1. Since 80% of req. sum = $400,
2. ∴ 1% of req. sum = $\frac{1}{80}$ of $400 = $5,
3. and 100% of req. sum = 100 × $5 = $500.

∴ ———

SHORT PROCESS:

$400 ÷ $\frac{80}{100}$ = $500. ∴ $500 is the req. sum.

Exercise XXXIII

1. 100% of a number is how many times the number?
2. 100% more than the cost = what % of the cost?
3. 25% less than $600 = what % of $600?
4. 125% of $300 = what % more than $300?
5. What number added to $16\frac{2}{3}$% of itself gives 77?
6. 40% of $500 is how many % of $\frac{3}{4}$ of $800?
7. $\frac{1}{2}$ is how many % less than $\frac{3}{4}$?
8. 564 is 20% more than what number?

9. What number added to 20% of itself gives 564?

10. $4.80 is 33⅓% more than what sum?

11. A man receives $1650 a year, and his expenses are 87½% of his income: how much has he left?

12. The number of children of school age in a certain city is 5220, which is 36% of the number of inhabitants: what is the population of the city?

13. A farm was sold for $6390, which was 12½% more than it cost. What was the cost of the farm?

14. A man sold a stock of goods for $10811, and gained 13⅘%. What was the cost of the goods?

15. A owns 42½% of a farm worth $35000, B owns 37% of it, and C owns the remainder. What is the value of each of their shares? Verify your work.

16. A farmer's crop of corn this year is 8% greater than his crop last year, and the two crops amounted to 5200 bushels. What was his last year's crop?

17. In the erection of a house, I paid twice as much for material as for labor. Had I paid 6% more for material, and 9% more for labor, my house would have cost $1284. What was its cost?

18. A clerk whose wages had been reduced 10% was receiving $63 per month. What were his wages before the reduction? Verify your work.

19. A man sold two horses for $360 each. On one he gained 25%, and on the other he lost 25%. Did he gain or lose, and how much?

20. Our stock decreased 33⅓%, and again 20%; then rose 20%, and again 33⅓%; we have thus lost $66: what was the stock worth at first?

APPLICATIONS OF PERCENTAGE — WITHOUT TIME

298. The **applications of percentage** may be divided into two classes: those in which *time is not considered,* and those in which *time is an element.*

In this chapter there will be considered only those applications of percentage in which the *time element* does not enter.

I. PROFIT AND LOSS

299. Fundamental Principle.—The gain or loss is reckoned at a certain per cent of the *cost* or *sum invested.*

EXAMPLES

1. A merchant sold a hat which cost $2.40, at a gain of 15%. What did he receive for it?

SOLUTION:

1. 100% of cost = $2.40, cost.
2. 1% of cost = $\frac{1}{100}$ of $2.40 = $0.024.
3. 15% of cost = 15 × $0.024 = $0.36, gain.
4. ∴ $2.40, cost, + $0.36, gain, = $2.76, amount received.

SHORT PROCESS:

$2.40 × .15 = $0.36, gain.
$2.40 + $0.36 = $2.76, amount received.

2. A hat that cost $4.50 was sold for $4.95. What was the gain per cent?

SOLUTION:

1. $4.95 − $4.50 = $0.45 = the gain.
2. $4.50 = $100% of the cost.
3. $0.01 = $\frac{1}{450}$ of 100% of the cost = $\frac{2}{9}\%$ of the cost.
4. $0.45 = 45 × $\frac{2}{9}\%$ of the cost = 10% of the cost.

Short Process.

$4.95 − $4.50 = $0.45 = the gain.
$0.45 ÷ $4.50 = .1 = 10%, *ans.*

Exercise XXXIV

1. A man sold cotton costing him $1500 at a gain of 16⅔%. Find the gain.

2. Mr. Smith bought goods and sold them at a loss of 12½%, losing $137. Find the selling price.

3. By selling a horse for $70, 16⅔% of its cost was gained. What did the horse cost?

4. A grain dealer sold ⅚ of his wheat for what the whole amount cost him. How many per cent did he gain?

5. A's home cost him $800; he sold it for $900; he bought it back for $1000. What % did he gain or lose?

6. A man bought cotton at 16⅔% less than the market price, and sold it at 20% more than the market price. What per cent did he gain?

7. A merchant sells hats for $5 apiece, making a profit of 33⅓%. What did the hats cost apiece?

8. John bought a pony for $25, and set a price so that after deducting $18 he gained 20%. What % of the asking price did he deduct?

9. If the selling price of an article is ⅚ of its cost, what per cent is lost?

10. A merchant bought a barrel of maple syrup, containing 46 gallons, at $2.50 per gallon. If 6 gallons leaked out, at how much per gallon must he sell the rest so as to gain 25%?

II. COMMERCIAL OR TRADE DISCOUNT

300. Commercial or **trade discount** is a deduction of a certain rate from bills, or from the list price of goods.

Manufacturers, publishers, and wholesale dealers usually issue catalogues in which the prices of their goods are listed. Trade discounts are made to avoid the necessity of changing the prices printed in the catalogues. As the cost of production varies, the market price changes accordingly, and the discounts are changed to meet the rise or fall in prices while the list price remains the same.

301. The discounts are reckoned at so many per cent. Frequently more than one discount is allowed.

Thus, "20 and 10 off" means 20% less than 100% of the list price, or 80% of the list price, and then 10% less than 80% of the list price, which is 72% of the list price.

The cost of an article at a discount of 20% and 10% off is 72% of the list price.

EXAMPLES

1. What is the cost of an article listed at $760, if the discounts are 20% and 15%?

SOLUTION: 1. $760 – 20% of $760 = $608.
2. $608 – 15% of $608 = $516.80, *ans.*

2. Find the cost of an article listed at $520, the discounts being 15% and 10%.

SOLUTION: $$\text{Cost} = \frac{\$520 \times 85 \times 90}{100 \times 100} = \$397.80, \textit{ ans.}$$

NOTE.—Show that this solution is virtually the same as the preceding one.

Exercise XXXV

1. The list price of a bill of goods is $450; if the discounts are 30%, 10%, and 5%, what is the cost?

2. If an article sells at 30 and 10 off for $6.30, find the list price.

3. Show that 30% and 20% off is equal to 44% off.

4. If an article is bought at a discount of 30% and 10%, and sold at list price, find the rate per cent profit.

5. A man bought goods listed at $250 at a discount of 10% and a certain per cent off for cash. If the cost was $213.75, what was the per cent off for cash?

6. A merchant buys goods listed at $2359 at discounts of 33⅓%, 10%, and 5%. Find the cost.

7. A man settled a bill of $20 with $13.50, the discounts being (—) and 10% off. Fill the blank.

8. A merchant sold an article at 20% and 10% off. If the discounts amounted to $5.60, for how much did the article sell?

9. What was the list price of the article in problem 8?

10. Show that 30%, 20%, and 5% off amounts to the same as 5%, 20%, and 30% off.

11. What is the difference on a bill of $650, between a discount of 30%, and a discount of 25% and 5% off?

12. How must I mark goods that cost me $450 so that I may sell at 10% off from marked price at a gain of 20%?

13. Find the difference between a direct discount of 30% and discounts of 15% and 15%.

14. A merchant paid $1323 for goods, and the discounts were 25%, 12½%, and 10%. Find the list price.

III. COMMISSION AND BROKERAGE

302. An **agent** is a person authorized to transact business for another. He may be a *commission merchant, broker, collector, factor, consignee,* or *correspondent.*

The person for whom the business is transacted is called the **principal.**

303. The **consignment** is the goods shipped.

304. Commission is the compensation allowed the agent for his services, and is some per cent of the money expended or collected.

305. Brokerage is the commission allowed a broker for his services. *Brokerage* is usually a much smaller per cent than *commission,* for the broker does not handle the goods.

306. The sum left after the commission and all other expenses have been deducted, is called the **net proceeds.**

307. Fundamental principles of commission:

1. If an agent sells, his commission is some per cent of the sales.
2. If an agent buys, his commission is some per cent of the purchase.

EXAMPLES

1. Find the net proceeds, if an agent has sold $348 worth of cotton at a commission of 5%.

Solution: 1. 100% of the sales = $348.
2. 1% " " " = $\frac{1}{100}$ of $348 = $3.48.
3. 5% " " " = 5 × $3.48 = $17.40.
4. $348 − $17.40 = $330.60, net proceeds.

2. An agent received $2160; after deducting his commission of 8% for buying, he bought wheat with what remained. How many dollars were spent for wheat?

Solution: 1. 108% of investment = $2160.
2. 1% " " = $\frac{1}{108}$ of $2160 = $20.
3. 100% " " = 100 × $20 = $2000.
∴ $2000 = the amount spent for wheat.

Exercise XXXVI

1. Find the net proceeds from the sale of 36 barrels of flour at $4 a barrel, commission 6¼%.

2. My principal instructed me to invest $1244.40 in wool after deducting my commission of 2%. What was my commission?

3. An agent sells 1200 barrels of apples at $4.50 a barrel, and charges 2½% commission. After deducting his commission of 8% for buying, he invests the net proceeds in cotton. What is his entire commission?

4. A lawyer charges $70 for collecting a debt of $1400. Find his per cent of commission.

5. An agent sold goods amounting to $4800, receiving 3¼% commission. After paying $25 charges, he invested the balance in cattle, charging 2½% commission on the investment. How much was invested?

6. Having sold a consignment of wheat on 3% commission, I am requested to invest the proceeds in hogs after deducting my purchase commission of 2%. My entire commission is $265. What did the hogs cost?

7. A lawyer having a debt of $785 to collect, compromises for 82%. What is his commission at 10%?

8. An agent sold a farm. After taking out his commission of 10%, he sent his principal $450. What did the purchaser pay for the farm?

9. A broker received $169.20 for selling a farm for $8460. What per cent was his brokerage?

10. My agent sold my flour at 4% commission; increasing the proceeds by $4.20, I bought wheat, paying 2% commission; wheat declining 3⅓%, my loss, including commissions, was $5: what was the flour worth?—R. N. H.,

IV. INSURANCE

308. Insurance is security guaranteed by one party to another against loss or damage. The two kinds of insurance are *personal* and *property.*

309. Personal insurance is a contract in which one party, in consideration of certain premiums received, agrees to pay a certain sum to the insured party or to his heirs or assigns, in the event of death, sickness, or serious accident. Personal insurance is either *life insurance, health insurance,* or *accident insurance.*

310. Property insurance is a guarantee of indemnity for loss sustained on account of any cause specified in the policy. The various kinds of property insurance are *fire, marine, live stock, plate glass,* etc.

311. The **policy** is the written contract between the insurer and the person insured.

312. The **premium** is the sum paid for insurance.

313. The **risk** is the total liability of the insurer in the event of total loss of property insured.

Note.—To protect themselves against fraud, insurance companies usually take risks at not more than two-thirds of the real value of the property insured. When the insured property suffers damage less than the amount of the policy, the insurance company pays only the estimated loss.

314. Fundamental principles of insurance:

1. In life insurance, the premium is usually a certain sum per $1000, payable annually, semi-annually, or quarterly.
2. In property insurance, the premium, which is usually paid annually, is some fixed per cent of the risk.

EXAMPLES

1. A merchant insured his store for $7500 at $2\frac{3}{4}\%$. What was the premium?

Solution: 1. 100% of the risk = $7500.
2. 1% " " " = $\frac{1}{100}$ of $7500 = $75.
3. $2\frac{3}{4}\%$ " " " = $2\frac{3}{4} \times \$75 = \$206.25$.
∴ the premium was $206.25.

2. If the premium is $57 and the policy is $2850, what is the rate?

Solution: 1. $2850 = the risk.
2. $1 = $\frac{1}{2850}$ of the risk.
3. $57 = 57 × $\frac{1}{2850}$ of risk = .02 of risk.
∴ the rate is .02, or 2%.

Exercise XXXVII

1. A man pays $12 for insuring his house for $800. What is the rate?

2. What will be the premium for insuring property, at $2\frac{3}{7}\%$, to cover $27370?

3. A shipment of cotton was insured at $4\frac{3}{8}\%$ for a sum equal to $\frac{5}{8}$ of its value. The premium was $122.50. What was the cotton worth?

4. A man obtained a life policy of $5000 at the rate of $10.70 per $1000. Find his annual premium.

5. A vessel valued at $50400 was insured for $\frac{3}{4}$ of its value at $1\frac{1}{2}\%$. The first company re-insured $\frac{2}{3}$ of the risk in a second company, paying $441 premium. Find the first premium, and the second rate of insurance.

6. Find the amount of the premiums paid in 15 years on a life policy of $6000, at $27.50 per $1000 annually.

7. If a man who is insured for $6000, at an annual premium of $31.40 per $1000, dies after 12 payments, how much more will his heirs get than has been paid in premiums?

8. Find the amount of the policy which will cover the value of a ship load of cotton worth $24822, and the premium at the rate of $1\frac{1}{2}\%$.

9. A man paid $42.30 for insuring $\frac{5}{8}$ of his house at $\frac{9}{10}\%$. What is the value of the house?

10. A man paid $21 to have his house insured for $\frac{3}{4}$ of its value at $1\frac{3}{4}\%$. What was the value of his house?

V. TAXES AND DUTIES

315. A **tax** is a sum of money levied on persons (poll tax) or on property (property tax).

316. A **poll tax** is a tax levied on each male citizen (unless exempted by law) without regard to his property.

Each person so taxed is called a *poll.*

317. A **property tax** is a tax levied on property, either *real estate* or *personal property.*

318. Real estate is immovable property, such as *houses, lands,* etc.

319. Personal property is movable property, such as *horses, money, furniture,* etc.

320. The **assessment roll** is the list prepared by the tax assessor, containing the names of all persons subject to taxation in that special district, the assessed valuation of their property, and the amount of their taxes.

NOTE.—In tax levies is almost the only place in business where the denomination *mill* is used. Because the tax rate is usually a very small per cent, it is rated at so many mills on the dollar. A rate of 3 mills on the dollar is the same as $\frac{3}{10}$ of 1%.

321. Duties or **customs** are taxes levied by the *national government* on imported goods. There are two kinds, *specific* and *ad valorem.*

322. A **specific duty** is a tax levied at a specified sum per *yard, ton, gallon,* etc.

323. An **ad valorem duty** (Latin *ad valorem,* according to value) is a tax levied at a certain per cent of the value of the goods.

324. Allowances are always made in collecting duties, in case of loss or damage. An allowance made for loss of liquids imported in bottles is called *breakage*, but if the loss is from barrels and casks, the loss is called *leakage*. An allowance made for the weight of bags, boxes, barrels, etc., is called *tare*. These allowances are made *before* levying duties.

NOTE 1.—In computing the duties, amounts under 50 cents are rejected ; those from 50 cents to a dollar are considered a dollar.

NOTE 2.—In this book, occupation taxes and stamp taxes for revenue and postal purposes are not considered.

EXAMPLES

1. In a certain county there are 7570 polls, and the property of the county is valued at $5897500. If the poll tax is $1.25 a head, and the rate on the property is ½% (5 mills), what is the total tax?

SOLUTION:

I. 1. Since the tax on one poll = $1.25,
   2. ∴ the tax on 7570 polls = 7570 × $1.25 = $9462.50.

II. 1. Since the tax on $1 = $0.005,
   2. ∴ the tax on $5897500 = 5897500 × $0.005 = $29487.50.

∴ $9462.50 + $29487.50 = $38950, total tax.

2. Find the duty on 440 dozen bottles of wine, duty 60 cents a dozen, and breakage 12½%.

SOLUTION:

I. 100% − 12½% = 87½%.

II. 1. Since 100% of invoice = 440 doz. bottles,
   2. ∴ 1% " " = 4.40 " "
   3. and 87½% " " = 385 " " .

III. 1. Since duty on 1 doz. = $0.60,
   2. ∴ duty on 385 doz. = $231.

Exercise XXXVIII

1. A tax of $6450 was levied upon the taxable values of a city, valued at $1935000. What was the rate?

2. A tax of $16500 is levied upon a certain city. The real estate amounts to $850000 and personal property to $350000. There are 500 polls, each taxed at $1. Find the rate.

3. In problem 2, how much is A's tax, who pays for 3 polls and whose taxable property is $8500?

4. A merchant imported 40 dozen bottles of champagne costing $15 per dozen. Find the duty at $5.50 per dozen, breakage being 5%.

5. What is the duty at 25% ad valorem, on 100 watches invoiced at 120 francs each, the custom-house valuation of the franc being $0.193?

6. Find the whole duty on 4500 pounds of wool, invoiced at 20 cents a pound, specific duty 10 cents per pound, ad valorem duty 12%, tare 8%.

7. Find the rate of the tax levy, when $17500 worth of property, and 40 polls at $1 each, produce a tax of $285.

8. What is the valuation of A's property, who pays $51 on a 15-mill levy?

9. The ad valorem duty on 1500 yards of Irish linen, valued at 40 cents a yard, is $210. Find the per cent of duty.

10. If it costs 2% to collect, and 4% of the tax is non-collectible, what is the amount of the revenue from a levy of $8500?

VI. STOCKS AND BONDS

325. **Stock** is the money or other property of railroads, banks, and other incorporated companies.

326. A **share** of stock is one of the equal parts into which the stock of the company is divided. The original value of each share is generally $100.

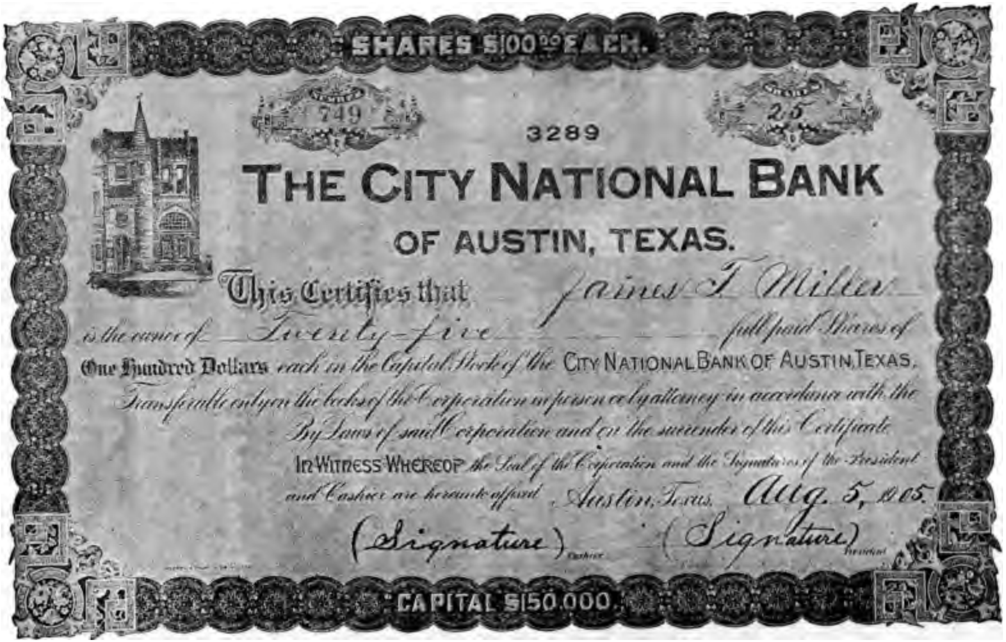
327. The **par value** of stock is its original, or face value, while its **market value** is the sum for which it will sell.

NOTE.—When a share will sell for its face value, it is said to be *at par;* when it will sell for more than its face value, it is said to be *above par*, or *at a premium;* when it sells for less than its face value, it is said to be *below par*, or *at a discount.* Stock quoted at 100 is at par ; at 108, is 8% above par; at 89, is 11% below par, etc.

328. A **certificate of stock** is a document showing the name of the company, the amount of its capital, the size of each share, the number of shares of the company, and the number of shares held by the person named in the certificate. Examine the form on page 152.

329. A **dividend** is a sum of money paid out of the earnings of the company to the stockholders. Dividends are declared when the earnings of the company are greater than the expenses.

330. An **assessment** is a sum required of the stockholders to meet the losses, or expenses of the company. Assessments are made when the earnings are less than the expenses.



SHARES $100.00 EACH.

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25

THE CITY NATIONAL BANK

OF AUSTIN, TEXAS.

This Certifies that James T. Miller

is the owner of Twenty-five full paid Shares of

One Hundred Dollars each in the Capital Stock of the CITY NATIONAL BANK OF AUSTIN, TEXAS.

Transferable only on the books of the Corporation in person or by attorney in accordance with the

By Laws of said Corporation and on the surrender of this Certificate.

IN WITNESS WHEREOF the Seal of the Corporation and the Signatures of the President

and Cashier are hereunto affixed Austin, Texas, Aug. 5, 1905.

(Signature) Cashier (Signature) President

CAPITAL $150,000.

FORM OF A STOCK CERTIFICATE

331. A **bond** is a certificate of indebtedness, given under the corporate seal, and signed by the proper officers. Bonds are simply notes of the incorporation, and they are issued for the purpose of borrowing money.

The principal bonds bought and sold as stock are government, state, county, city, and railroad bonds. Bonds are quoted according to the rate of interest which they bear.

Thus, U. S. 4's are bonds issued by the United States, bearing 4% interest.

332. A **stock broker** is an agent whose business it is to buy and sell stocks. He charges *brokerage*, usually $\frac{1}{8}$% of the *par value.* This is charged for buying and also for selling.

333. Fundamental principles:

1. Brokerage, interest on bonds, assessments, and the rate of dividends are always reckoned on the *par value.*
2. The rate of income is always reckoned on the *cost* (market value plus brokerage).

EXAMPLES

1. Find the cost of 110 shares of railroad stock, the market value of which is 103$\frac{3}{4}$, brokerage $\frac{1}{4}$%.

SOLUTION:

1. Market value of 1 share = $103.75
2. Brokerage on 1 share = $0.25
3. Cost of 1 share = $104.00
4. Cost of 110 shares = 110 × $104 = $11440, *ans.*

NOTE.—Unless otherwise mentioned, the par value of one share is $100.

2. How many shares of stock, at a discount of 11½%, can be bought with $10635, brokerage ⅛% ?

SOLUTION:

I. 1. Market value of 1 share = $88.50
   2. Brokerage on 1 share = $ 0.125
   3. Cost of 1 share = $88.625

II. 1. Since $88.625 will buy 1 share,
   2. ∴ $1 will buy $\frac{1}{88.625}$ of 1 share,
   3. and $10635 will buy $10635 \times \frac{1}{88.625}$ of 1 share, or 120 shares, *ans.*

3. At what price must stock be bought that pays 6% dividends, so as to realize an income of 7½% on the investment?

SOLUTION:

1. 7½ % of market value = 6% of par val.
2. 1 % " " " $= \frac{1}{7.5}$ of 6%, or $\frac{6}{7.5}$ % of par value.
3. 100% " " " $=100 \times \frac{6}{7.5}$ %, 80% of par value.
4. ∴ the stock must be bought for 80.

4. Find the income from $4880 invested in 4% bonds purchased at 121⅞%, brokerage ⅛%.

SOLUTION:

I. 1. $121⅞ + $⅛ = $122, cost of 1 bond.
   2. $4880 ÷ $122 = 40, number of bonds bought.

II. 1. Since 1 bond yields $4,
   2. ∴ 40 bonds yield 40 × $4 = $160, *ans.*

5. What rate per cent will be realized from 10% bonds bought at 125?

Solution:

1. $125 invested produces $10 income.

2. $1 invested produces $\frac{1}{125}$ of $10, or 8¢ income.

3. ∴ the rate of income is 8%.

Exercise XXXIX

1. What must be paid for 160 shares of bank stock at 12¼% premium, brokerage ⅛% ?

2. Find the cost of 20 shares of railroad stock at 112¾%, brokerage ⅛%.

3. City bonds are at a discount of 5%, but pay 3% interest. If A invests $6685 in such bonds, brokerage ½%, what will be his yearly income?

4. What must be invested in Tenn. 6's at 85, to yield an income of $1800?

5. What per cent of income do U. S. 4's yield, if bought at a premium of 19½%, brokerage ½%?

6. What sum must be invested in 3% bonds at 96½, brokerage ⅛%, to yield an annual income of $180?

7. Which will yield the better income, 8% bonds at 110, or 5% bonds at 75?

8. What must be paid for 6% bonds that the investment may yield 8%?

9. How much must be invested, if stock 20% below par yield a 6% income of $390?

10. A buys 100 shares of railroad stock at 118⅛, brokerage ⅛, keeps it one year, receives a 5% dividend, sells it at 138⅝, brokerage ⅛. If money is worth 4% a year, what does A gain?

11. The trustees of a college had $48000, and invested one-half of it in 5% railroad bonds at 95⅞, and the other half in 6% city bonds at 119⅞, paying ⅛% brokerage in each case. What income did they secure?

12. Find the difference in dollars between the income on $1500 invested in 6% stock at 75, and the income on the same sum invested in 4% stock at 60.

13. How much money must be invested in railroad stock paying a 5% dividend, at 92⅝, to secure an annual income of $1000?

14. A broker, who charges ⅛%, received $54.75 for selling stock. How many shares did he sell?

15. How many $100 U. S. 5's at 103⅞ can be purchased with $5000, brokerage ⅛%? What is the uninvested balance?

16. The net earnings of a gas company are $35000 and the capital stock is $650000. If a surplus of $2500 is reserved, what rate of dividend can be declared?

17. A stock company with a capital of $527000 loses during a certain year $7905. If the stockholders are assessed to pay this loss, what will be the rate of assessment, and how much will A pay, who owns 18 shares?

18. Which is the better investment, U. S. 4½'s, or Tenn. 6's, both at par, when taxes on personal property average 2%?

NOTE.—United States bonds are not subject to taxation, but all other bonds are assessed as other personal property.

19. If a man sells $15600 worth of city bonds at 97, and invests a sufficient amount of the proceeds in U. S. 4's, at

107, to yield an income of \$360 annually, and purchases a farm with the remainder, how much will the farm cost him?

20. A man owned a farm of 160 acres, which annually yielded a net income of \$6 per acre. He sold the farm at \$75 an acre, and invested the proceeds in 8% stock at 160. What is the change in his annual income?

CHAPTER XII

APPLICATIONS OF PERCENTAGE—WITH TIME

I. SIMPLE INTEREST

334. In this chapter the applications of percentage contain the *element of time.*

335. Interest is money charged for the use of money.

336. The **principal** is the money for the use of which interest is charged.

337. The **time** is the period during which the principal bears interest.

338. The **amount** is the sum of the principal and the interest.

339. The **rate of interest** is the per cent to be taken of the principal for one interval of time.

NOTE 1.—The *rate per cent* means the *rate per cent per annum* unless some other period of time is specified.

NOTE 2.—The *legal rate* is the rate established by law, and is always understood when no rate is specified in a contract.

The *contract rate* is the maximum rate permitted by law when specified in a contract. Legal and contract rates vary.

Usury is any rate *higher* than the contract rate.

340. Simple interest is the interest on the principal only. It is found by taking the continued product of the *principal by the rate by the time expressed in years.* Using the initial letters, we have the formula: I = P × R × T.

NOTE.—In the following table, *G* represents 3 days of grace; *P*, previous business day; *S*, succeeding business day.

States and Territories.	INTEREST LAWS.				STATUTES OF LIMITATION.		
	Legal Rate.	Maxim. Contract Rate.	Grace.	Payable.	Judgments.	Notes.	Open Accounts
					Years.	*Years.*	*Years.*
Alabama	8	8	G	S	20	6	3
Arkansas	6	10	G	P	10	5	3
Arizona	6	Any		S	5	4	3
California	7	Any		S	5	4	2
Colorado	8	Any		S	20	6	6
Connecticut	6	6		S			6
Delaware	6	6		P	10	6	3
Dist. of Columbia	6	10		S	12	3	3
Florida	8	10		S	20	5	2
Georgia	7	8		P	7	6	4
Idaho	7	12		S	6	5	4
Illinois	5	7		S	20	10	5
Indiana	6	8	G	P	20	10	6
Iowa	6	8		S	20	10	5
Kansas	6	10	G	P	5	5	3
Kentucky	6	6	G	P	15	15	2
Louisiana	5	8		S	10	5	3
Maine	6	Any		S	20	6	6
Maryland	6	6		S	12	3	3
Massachusetts	6	Any		S	20	6	6
Michigan	5	7	G	S	6	6	6
Minnesota	7	10		P	10	6	6
Mississippi	6	10	G	P	7	6	3
Missouri	6	8	G	S	10	10	5
Montana	8	Any		S	10	8	5
Nebraska	7	10	G	S	5	5	4
Nevada	7	Any	G	P	6	4	4
New Hampshire	6	6		S	20	6	6
New Jersey	6	6		S	20	6	6
New Mexico	6	12	G	S	7	6	4
New York	6	6		S	20	6	6
North Carolina	6	6	G	S	10	3	3
North Dakota	7	12		S	10	6	6
Ohio	6	8		S	15	15	6
Oklahoma	7	12	G	S	5	5	3
Oregon	6	10		S	10	6	6
Pennsylvania	6	6		S	5	6	6
Rhode Island	6	Any		S	20	6	6
South Carolina	7	8	G	S	20	6	6
South Dakota	7	12	G	S	10	6	6
Tennessee	6	6		S	10	6	6
Texas	6	10	G	P	10	4	2
Utah	8	Any		S	8	6	4
Vermont	6	6		S	8	6	6
Virginia	6	6		S	20	5	2
Washington	6	12		S	6	6	3
West Virginia	6	6		S	10	10	3
Wisconsin	6	10		S	20	6	6
Wyoming	8	12	G	P	5	5	3

EXAMPLES

1. Find the interest of \$450 for 120 days at 8%.

SOLUTION BY FORMULA:

$$\text{Since } I = P \times R \times T, \therefore I = \$450 \times \tfrac{8}{100} \times \tfrac{120}{360} = \$12.$$

SOLUTION BY ANALYSIS:

$$\text{I.}\begin{cases} \text{1. Since } 100\% \text{ of principal} = \$450, \\ \text{2. } \therefore \quad 1\% \text{ '' \quad '' } = \tfrac{1}{100} \text{ of } \$450 = \$4.50. \\ \text{3. and } \quad 8\% \text{ '' \quad '' } = 8 \times \$4.50 = \$36, = \text{int. for 1 yr.} \end{cases}$$

$$\text{II.}\begin{cases} \text{1. Since int. for 360 days} = \$36, \\ \text{2. } \therefore \quad \text{int. for \quad 1 day} = \tfrac{1}{360} \text{ of } \$36 = \$0.1, \\ \text{3. and \quad int. for 120 days} = 120 \times \$0.1 = \$12. \end{cases}$$

NOTE.—In ordinary business transactions, 30 days are considered a month, and 360 days a year.

Using the formula above, make a formula for finding the *principal, the rate*, and the *time. Write the rule* for each formula made.

2. At what rate will \$360 produce \$24 interest in 1 year and 4 months?

SOLUTION:

$$\text{Since } I = P \times R \times T, \quad \therefore R = \frac{I}{P \times T}.$$

$$\text{Substituting: } R = \frac{\$24}{\$360 \times 1\tfrac{1}{3}} = .05, \text{ or } 5\%.$$

ANALYSIS:

$$\text{I.}\begin{cases} \text{1. Since int. for 1 yr. 4 mo., or } \tfrac{4}{3} \text{ yr.,} = \$24, \\ \text{2. } \therefore \quad \text{int. for 1 yr.} = \tfrac{3}{4} \text{ of } \$24 = \$18. \end{cases}$$

$$\text{II.}\begin{cases} \text{1. Since } \$360 = 100\% \text{ of the principal,} \\ \text{2. } \therefore \quad \$1 = \tfrac{1}{360} \text{ of } 100\% = \tfrac{5}{18}\% \text{ of the principal,} \\ \text{3. and } \quad \$18 = 18 \times \tfrac{5}{18}\%, \text{ or } 5\% \text{ of the principal.} \end{cases}$$

$\therefore$ the rate = 5%.

3. In what time will $480 produce $60 at 5%?

SOLUTION:

Since $T = \frac{I}{P \times R}$, ∴ $T = \frac{\$60}{\$480 \times .05} = 2\frac{1}{2}$, or $2\frac{1}{2}$ yr.

NOTE.—The solution by the formula is purely mechanical. It will do for commercial purposes, but for educational purposes the solution by analysis is far more valuable.

ANALYSIS:

I.
1. Since 100% of prin. = $480,
2. ∴ 1% " " = $\frac{1}{100}$ of $480 = $4.80,
3. and 5% " " = 5 × $4.80 = $24.

II.
1. Since $24 is the int. for 1 yr.,
2. ∴ $1 " " " " $\frac{1}{24}$ yr.,
3. and $60 " " " " 60 × $\frac{1}{24}$ yr., or $2\frac{1}{2}$ yr.

4. What principal will amount to $2394 in 2 years and 4 months at 6%?

SOLUTION:

Assume $1 as a principal.

I.
1. Since 100% of prin. = $1,
2. ∴ 1% " " = $\frac{1}{100}$ of $1 = $0.01,
3. and 6% " " = 6 × $0.01 = $0.06.

II.
1. Since the int. for 1 yr. = $0.06,
2. ∴ the int. for $\frac{7}{3}$ yr. = $\frac{7}{3}$ × $0.06 = $0.14.
3. Now, $1 + $0.14 = $1.14, am't of $1 for 2 yr. 4 mo.

III.
1. Since $1.14 am't requires $1 prin.,
2. ∴ $1 " " $\$\frac{1}{1.14}$ prin.,
3. and $2394 " " 2394 × $\$\frac{1}{1.14}$ prin. = $2100 prin.

SHORT PROCESS:

1. Int. on $1 for 2 yr. 4 mo. at 6% = $0.14.
2. Am't of $1 for 2 yr. 4 mo. at 6% = $1.14.
3. $2394 ÷ $1.14 = 2100. ∴ $2100 is the prin.

NOTE.—By means of the solution above, the ***present worth*** of a debt is found.

341. The **present worth** of a debt payable at a future date without interest is that sum which, when put on interest for the time yet to elapse, will amount to the debt.

The above example might be stated thus: What is the *present worth* of $2394, due 2 yr. 4 mo. hence, money being worth 6%, simple interest?

342. The **true discount** is the difference between the debt and the present worth. It is, therefore, the simple interest on the present worth.

5. Find the present worth and true discount of a debt of $627 due in 9 months, money being worth 6%.

SOLUTION:

$$\text{Present worth} = \frac{A}{1 + R \times T} = \frac{\$627}{1 + \frac{6}{100} \times \frac{9}{12}} = \frac{\$627}{\frac{209}{200}} = \$600.$$

$$\$627 - \$600 = \$27, \text{ true discount.}$$

The meaning is: $600 put on interest at 6% for 9 mo. would amount to $627.

Solve this problem by analysis, following the form in example 4 above.

NOTE.—In *true discount*, it is not customary to allow 3 days of grace.

343. In **bankers' interest** (bank discount), the exact number of days is counted; but, in expressing the time in years, 360 days is considered 1 year. Thus,
the time from May 2, 1904, to August 5, 1904, is 95 days, or $\frac{95}{360}$ yr.

344. In **exact interest**, the exact number of days is counted; but, in expressing the time in years, 365 days is considered 1 year (366 days in leap years). Thus,
the time from January 1, 1904, to July 12, 1904, is 193 days, or $\frac{193}{366}$ yr.

Exercise XL

Find the simple interest on:

1. $370 for 2 years at 5%.
2. $890 for 1 year 4 months at 9%.
3. $1342 for 8 months at 10%.
4. $1 for 30 days at 6%.
5. $1 for 1 day at 1%.

Find the amount of:

6. $1568 for 5 yr. 8 mo. 18 da. at 8%.
7. $4698.58 for 6 yr. 7 mo. 25 da. at 5½%.

Find the rate required for:

8. $400 to produce $57 int. in 1 yr. 7 mo.
9. $900 to produce $25.20 int. in 8 mo. 12 da.

Find the time required for:

10. $500 to produce $55 interest at 8%.
11. $2500 to produce $131.25 interest at 7%.

Find the principal required to produce:

12. $25.20 interest in 1 yr. 9 mo. at 4%.
13. $35 interest in 3 mo. at 14%.

Find the principal which amounts to:

14. $751.20 in 6 mo. 15 da. at 8%.
15. $2516.66⅔ in 30 da. at 8%.
16. $240.50 from Sept. 1 to Dec. 31, at 4%.

Find the interest (bankers' interest method) on:

17. $385 from Aug. 12, 1891. to Nov. 1, 1891, at 12%.
18. $840 from May 12, 1904, to Sept. 5, 1904, at 8%.

Find the exact interest on:

19. $850 from Jan. 15, 1904, to June 20, 1904, at 8%.

20. $125 from Jan. 20, 1905, to Nov. 19, 1905, at 6%.

21. On what date, counting from Jan. 15, will $200 at 6% produce as much accurate interest as it would ordinary interest from Jan. 15 to Sept. 15?

Miscellaneous:

22. What principal at 6% will amount to $450 in 5 years?

23. Mr. Roe borrowed $1575 on the 1st of April. On the 1st of November following he paid the amount, which was $1630.125. What rate of interest did he pay?

24. In what time will any principal double itself at 8%?

25. At what rate will any principal double itself in 12 years?

26. What is the difference between the true discount of $248.76, due in 2 yr. 3 mo. 15 da., and the simple interest on the same amount for the same time, money being worth 6%?

27. A note of $87.50, dated Aug. 8, 1901, and bearing interest at 10%, was paid March 25, 1903. What was the amount paid?

28. Find the amount of $63.96 for 1 yr. 7 mo. 10 da., at $6\frac{1}{4}$%.

29. What is the interest on a note of $600, dated June 10, 1900, and paid Jan. 3, 1901, at 5% interest?

30. A man who has $1650 in a savings bank receives $41.25 every 6 months. What per cent interest does he receive on his money?

THREE IMPORTANT RULES:

1. **To find the simple interest:**

   Multiply the principal by the rate and the result by the time expressed in years, and the result will be the simple interest.

2. **To find the present worth:**

   Divide the debt by the amount of $1 *for the given time and rate, and the quotient will be the present worth.*

3. **To find the true discount:**

   Subtract the present worth from the debt, and the remainder will be the true discount.

II. BANK DISCOUNT

345. A **bank** is an institution authorized by law for the safe-keeping, loaning, and issuing of money.

346. Bank discount is the sum deducted by a bank for paying a note or draft before it is due, or the sum charged for lending money. *It is simple interest on the maturity value of a note or draft for the time it has yet to run.*

347. The **term of discount** is the time from the date of discount to, and including, the date of maturity.

348. The **date of maturity** is the last day of grace (in states allowing 3 days of grace) or the day on which the note is legally due.

NOTE 1.—Many of the states have abolished the law allowing 3 days of grace. For your own state, see table, p. 159.

NOTE 2.—Notes falling due on a legal holiday, or on Sunday, mature and are payable on the preceding or succeeding business day, according to the law or custom of the locality. See p. 159.

349. The **proceeds** (called also *net proceeds*) of a note or draft is the difference between the *maturity value* and the *bank discount.*

NOTE.—In bank discount, 3 days of grace are considered in this book.

EXAMPLES

1. Find the bank discount and proceeds of a note of $500, due in 90 days, discounted at 8%.

SOLUTION:

1. The term of discount = 93 days.
2. $\$500 \times \frac{8}{100} \times \frac{93}{360} = \$10.33\frac{1}{3}$ = bank discount.
3. $\$500 - \$10.33\frac{1}{3} = \$489.66\frac{2}{3}$ = proceeds.

Step 2 may be analyzed as in example 1 in simple interest.

2. I wish to borrow $1800 for 90 days. For what sum must the note be made, if the bank discounts it at 8%?

SOLUTION:

1. The discount on $1 for 93 days = $\$0.02\frac{1}{15}$.
2. The proceeds on $1 for 93 days = $\$0.97\frac{14}{15}$.
3. $\$1800 \div \$0.97\frac{14}{15} = 1837.985$.
4. ∴ the note must be made for $1837.985.

3. $1100. MARLIN, TEX., *July* 5, 1904.

Three months after date, I promise to pay David Doe, or order, eleven hundred dollars, at the Merchants' National Bank, with interest at 6%. Value received.

JOHN ROE.

Discounted Aug. 5, 1904.

SOLUTION:

1. This note matures Oct. 5 + 3 days, or Oct. 8.
2. The int. on $1100 for 93 days at 6% = $17.05.

3. $1100 + $17.05 = $1117.05 = the *maturity value.*
4. The *term of discount* is from Aug. 5 to Oct. 8, or 64 days.
5. The int. on $1117.05 for 64 days at 6%=$11.92=bank discount.
6. $1117.05 – $11.92 = $1105.13, the proceeds.

Exercise XLI

Find the bank discount and proceeds of a note:

1. Of $386, due in 100 days, discounted at 6%.

2. Of $36.20, payable in 80 days, discounted at 5%.

3. Of $850, payable in 90 days, discounted at 7%.

4. A merchant is offered $3000 worth of goods on 3 months' time, or 5% off for cash. If he borrows the money at the bank, at 7%, and pays cash, will he gain or lose, and how much?

5. A man borrows $1000 for 60 days; bank discount is 8%. Find the face of the note.

6. A note of $600 dated July 28, at 3 months, was discounted at bank Sept. 5, at 8%. Find the proceeds.

7. $1200. SPRINGFIELD, ILL., *Aug.* 5, 1903.

> Ninety days after date, I promise to pay Tom Johnson, or order, twelve hundred dollars, for value received, at the City National Bank.
>
> BILL WILLIAMS.

Find the proceeds, if discounted Sept. 2, 1903, at 8%.

8. $640. MACON, GA., *March* 10, 1903.

> Five months after date, I promise to pay to the order of Henry Babb, six hundred forty dollars, for value received, with interest at 6%.
>
> M. P. RYE.

Find the proceeds, if this note was discounted at the bank, at 8%, May 29, 1903.

9. My note of $360, which will be legally due in 1 yr. 4 mo. 20 da., is discounted at a bank, at 8%. What rate of interest does the banker receive?

NOTE.—**The difference between true discount and bank discount is equal to the simple interest on the true discount for the given time at the given rate.** Illustrate this fact by solving the following example:

10. Find the difference between the true discount and the bank discount on $440, due in 1 yr. 8 mo., at 6%, not reckoning days of grace.

III. ANNUAL INTEREST

350. Annual interest is simple interest on the principal, augmented by the simple interest on each interval's interest from the close of the interval to the time of settlement.

EXAMPLE

A note of $600.00 drawing 6% interest, payable annually, runs for 4 years and 3 months. Find the amount due at maturity, if no payments have been made.

SOLUTION:

Each year's interest will bear interest after it becomes due, as follows:

I.
1. 1st year's interest for 3 years, 3 months.
2. 2d year's interest for 2 years, 3 months.
3. 3d year's interest for 1 year, 3 months.
4. 4th year's interest for 0 years, 3 months.

Total = 7 years.

To the simple interest on the principal for 4 years, 3 months, we must add the interest on 1 year's interest for the sum of the above intervals, 7 years.

II.
1. Int. on $600 for 1 yr. at 6% = $36, due annually.
2. Int. on $600 for 4¼ yr. at 6% = $153, int. on prin.
3. Int. on $36 for 7 yr. at 6% = $15.12, int. on int.
4. $600 + $153 + $15.12 = $768.12, amount due.

Exercise XLII

1. A note of $350 drawing 5% interest, payable annually, runs 4 years, 6 months. Find the amount due at maturity, if no payments have been made.

2. A man bought $500 worth of goods on 9 months' credit; he paid for them at the end of 3 years, 3 months. Allowing 6% interest, payable annually, how much was due?

3. A gentleman holds six $1000 railroad bonds, due in 3 years, interest 6% payable semi-annually: no interest having been paid, what amount is owing him when the bonds mature?—R. N. H., p. 260.

NOTE.—Frequently annual or periodic interest is payable semi-annually or quarterly.

4. Find the annual interest on $800 for 1 year, 9 months, at 5% per annum, payable quarterly.

5. The annual interest on a sum of money for 4 years, 6 months, at 5%, paid annually, was $85.75. What was the sum?

SUGGESTION.—Find the annual interest on $1.

6. Find the amount due at the end of 5 years, 9 months,

of a note of $1200, bearing interest at 6%, payable annually, if no payments have been made.

Remark.—In some states, the law provides that unpaid annual interest shall bear interest at the legal rate. In the following problem, the unpaid interest bears the legal rate of interest.

7. In a state whose legal rate is 6%, H makes a note of $1250 for 3 yr. 3 mo., with interest at 8%, payable semiannually; he pays no interest; find the amount due at maturity.—S. & K., p. 197.

IV. COMPOUND INTEREST

351. Compound interest is the interest that accrues by making the interest, due at the close of any interval, a part of the interest-bearing principal for the next succeeding interval.

EXAMPLE

Find the compound interest on $1 for 3 years at 5%, payable annually.

Solution:

$$
\begin{array}{rl}
\$1.00 & = \text{principal } (P). \\
.05 & = \text{rate } (R). \\ \cline{1-1}
\$0.05 & = \text{int. for 1st yr.} \\
1.00 & \\ \cline{1-1}
\$1.05 & = (1 + R) = \text{amount for 1st yr. } (A). \\
.05 & \\ \cline{1-1}
\$0.0525 & = \text{int. for 2d yr.} \\
1.05 & \\ \cline{1-1}
\$1.1025 & = (1 + R)^2 = \text{amount for 2d yr.} \\
.05 & \\ \cline{1-1}
\$0.055125 & = \text{int. for 3d yr.} \\
1.1025 & \\ \cline{1-1}
\$1.157625 & = (1 + R)^3 = \text{amount for 3d yr.} \\
1. & = \text{the original principal.} \\ \cline{1-1}
\$0.157625 & = \text{the compound int. for 3 years.}
\end{array}
$$

A careful study of this example will show that the compound amount of $1, at any rate, for any number of years, equals (1 + R) raised to the power indicated by the number of years. Then to get the amount of *n* dollars, take *n* times the amount of $1.

Using the initial letters, we have the following formulas:

$$\text{I. } A = P(1 + R)^T$$

$$\text{II. } P = \frac{A}{(1 + R)^T}$$

$$\text{III. } I = A - P$$

Write the rule **corresponding to each formula.**

352. The **interest interval** is generally a *year, half-year,* or *quarter-year.* The length of the interval is indicated by inserting *annually, semi-annually,* or *quarterly.*

The *half-interval* rate $= \sqrt{1 + \text{interval rate}} - 1$.

Thus, if the annual rate is 21%, the *semi-annual rate* would $= \sqrt{1.21} - 1 = 1.10 - 1 = .10$, or 10%.

This fact may be illustrated thus:

1. Am't of $100 for 1 yr. at 21% annually = 1.21 × $100 = $121.

2. Am't at 21% semi-annually = $\sqrt{1.21} \times \sqrt{1.21} \times \$100 = \$121$.

3. ∴ the semi-annual *multiplier* = $\sqrt{1.21}$, and the rate = $\sqrt{1.21} - 1$.

Similarly, it can be shown that the quarterly multiplier = $\sqrt[4]{1.21}$, and the rate = $\sqrt[4]{1.21} - 1$.

TABLE

Showing the amount of $1 at compound interest from 1 year to 20 years.

Yr.	2½ per cent.	3 per cent.	3½ per cent.	4 per cent.	5 per cent.	6 per cent.
1	1.025	1.03	1.035	1.04	1.05	1.06
2	1.050625	1.0609	1.071225	1.0816	1.1025	1.1236
3	1.076891	1.092727	1.108718	1.124864	1.157625	1.191016
4	1.103813	1.125509	1.147523	1.169859	1.215506	1.262477
5	1.131408	1.159274	1.187686	1.216653	1.276282	1.338226
6	1.159693	1.194052	1.229255	1.265319	1.340096	1.418519
7	1.188686	1.229874	1.272279	1.315932	1.4071	1.50363
8	1.218403	1.26677	1.316809	1.368569	1.477455	1.593848
9	1.248863	1.304773	1.362897	1.423312	1.551328	1.689479
10	1.280085	1.343916	1.410599	1.480244	1.628895	1.790848
11	1.312087	1.384234	1.45997	1.539454	1.710339	1.898299
12	1.344889	1.425761	1.511069	1.601032	1.795856	2.012197
13	1.378511	1.468534	1.563956	1.665074	1.885649	2.132928
14	1.412974	1.51259	1.618695	1.731676	1.979932	2.260904
15	1.448298	1.557967	1.675349	1.800944	2.078928	2.396558
16	1.484506	1.604706	1.733986	1.872981	2.182875	2.540352
17	1.521618	1.652848	1.794676	1.947901	2.292018	2.692773
18	1.559659	1.702433	1.857489	2.025817	2.406619	2.854339
19	1.59865	1.753506	1.922501	2.106849	2.52695	3.0256
20	1.638616	1.806111	1.989789	2.191123	2.653298	3.207136

Yr.	7 per cent.	8 per cent.	9 per cent.	10 per cent.	11 per cent.	12 per cent.
1	1.07	1.08	1.09	1.10	1.11	1.12
2	1.1449	1.1664	1.1881	1.21	1.2321	1.2544
3	1.225043	1.259712	1.295029	1.331	1.367631	1.404908
4	1.310796	1.360489	1.411582	1.4641	1.51807	1.573519
5	1.402552	1.469328	1.538624	1.61051	1.685058	1.762342
6	1.50073	1.586874	1.6771	1.771561	1.870414	1.973822
7	1.605781	1.713824	1.828039	1.948717	2.07616	2.210681
8	1.718186	1.85093	1.992563	2.143589	2.304537	2.475963
9	1.838459	1.999005	2.171893	2.357948	2.558036	2.773078
10	1.967151	2.158925	2.367364	2.593742	2.83942	3.105848
11	2.104852	2.331639	2.580426	2.853117	3.151757	3.478549
12	2.252192	2.51817	2.812665	3.138428	3.49845	3.895975
13	2.409845	2.719624	3.065805	3.452271	3.883279	4.363492
14	2.578534	2.937194	3.341727	3.797498	4.31044	4.887111
15	2.759031	3.172169	3.642482	4.177248	4.784588	5.473565
16	2.952164	3.425943	3.970306	4.594973	5.310893	6.130392
17	3.158815	3.700018	4.327633	5.05447	5.895091	6.86604
18	3.379932	3.996019	4.71712	5.559917	6.543551	7.689964
19	3.616527	4.315701	5.141661	6.115909	7.263342	8.61276
20	*3.869684*	4.660957	5.604411	6.7275	8.062309	9.646291

Exercise XLIII

Find the compound interest on:

1. \$500 for 4 years at 5%, payable annually.
2. \$1700 for 2 years at 4%, payable semi-annually.
3. \$450 for 5 yrs., 4 mos., at 6%, payable semi-annually.

Find the compound amount on:

4. \$800 for 2 years at 6%, compounded semi-annually.
5. \$1848 for 5 years, 4 months, 15 days, at 7%, compounded annually.
6. Find the difference between the simple and the compound interest on \$180 for 2 years at 8%, if the interest is compounded quarterly.
7. What principal will amount to \$763.205 in 2 years, 9 months, at 6%, compounded annually?
8. Find the compound interest on \$500 for 3 years, 2 months, at 6%, compounded annually.

SOLUTION:

By referring to the table (page 172), it will be seen that the amount of \$1 for 3 yr. at 6% = \$1.191016.

Computing the interest on this amount for 2 mo., the amount of \$1 for 3 yr. 2 mo. is \$1.202926. Therefore, the amount of \$500 = 500 × \$1.202926 = \$601.463. \$601.463 − \$500 = \$101.463, compound interest.

Making use of the table, find the compound interest on:

9. \$535 for 3 yr. 5 mo. at 7%.
10. \$750.80 for 6 yr. 7 mo. at 6%.
11. \$672.28 for 2 yr. 3 mo. 18 da. at 6%.
12. What rate per cent per annum, compounded semi-annually, is equivalent to 44%, compounded annually?

V. PARTIAL PAYMENTS

353. A **partial payment** is a payment of part of a debt.

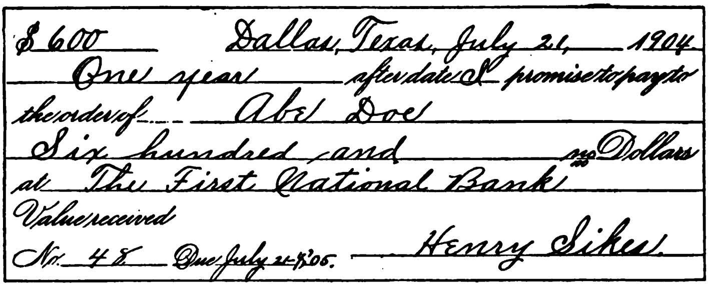
Payment on a note, or other interest-bearing obligation, is frequently made before it is due, to lessen the interest.

354. A **note,** or **promissory note,** is a written promise to pay a specified sum at a specified time, or on demand.

355. The **maker,** or **drawer,** or **promisor,** of a note is the person who signs it, and who thereby promises to pay it.

356. The **payee** is the person to whom, or to whose order, the note is made payable.

357. The **face** is the sum specified in the note.



$600 Dallas, Texas, July 21, 1904
One year after date I promise to pay to
the order of Abe Doe
Six hundred and no/00 Dollars
at The First National Bank
Value received
No. 48 Due July 21-24 '05. Henry Sikes

In this promissory note, Henry Sikes is the *maker,* Abe Doe the *payee,* $600 the *face,* and the face plus the interest the *maturity value.*

358. A **negotiable note** is one made payable to *bearer,* or to the *order* of the payee.

A negotiable note may be sold by the payee, the transfer being indicated by the payee's *indorsing* the note; that is, by writing his name across the back of the note. By indorsing the note, the payee becomes responsible for its payment in case the maker does not pay it. The indorser, however, may be released by first writing the words "without recourse" across the back, and then his name.

359. If a note reads "with interest," but does not specify the rate, it draws the legal rate. If it does not call for interest, it draws none until it becomes due and payment is demanded, after which it draws the legal rate.

360. A **protest** is a formal declaration in writing, made by a notary public, at the request of the holder of a note, notifying the maker and the indorsers of its non-payment.

NOTE.—The failure to protest a note on the day of maturity or immediately thereafter releases the indorsers from all obligation to pay it, unless the note contains the words "*protest waived.*"

361. The **United States Rule** is most generally used in making computations involving partial payments. This rule was first announced by Chancellor Kent of New York. It has been adopted by the Supreme Court of the United States, and by the courts of nearly all the states.

UNITED STATES RULE.—1. *Find the amount of the given principal to the time of the first payment, and if this payment equals or exceeds the interest then due, subtract it from the amount obtained, and treat the remainder as a new principal.*

2. *But if the interest is greater than any payment, find the amount on the same principal to a time when the sum of the payments equals or exceeds the interest then due; subtract the sum of the payments from that amount, regard the remainder as a new principal, and proceed as before.*

362. **Fundamental principles** of the U. S. Rule:

1. Making a payment makes the interest on the principal fall due, up to that time.

2. A partial payment must be applied first to the discharge of the interest due, and then the balance, if any, to the discharge of the principal.
3. Interest must not bear interest.
4. Payments must not bear interest.

EXAMPLE

A note of $800 is dated Oct. 1, 1904, with interest at 8%. This note has the following indorsements:

Jan. 1, 1905, $80; April 19, 1905, $10; Sept. 1, 1905, $240; what was due Jan. 1, 1906?

Dates.
1906— 1— 1
1905— 9— 1
1905— 4—19
1905— 1— 1
1904—10— 1 *Payments.*
3— 0.... $80.
3—18.... $10.
4—12....$240.
4— 0

Note.—Beginning with the latest date above, arrange the dates in order. Beginning below, subtract each date from the one immediately above it.

Solution:

1. Int. on $800 for 3 mo. at 8% = $16.
2. $800 + $16 – $80 = $736, 2d, or new principal.
3. Int. on $736 for 3 mo. 18 days at 8% = $17.66.
4. $17.66 – $10 = $7.66, unpaid interest.
5. Int. on $736 for 4 mo. 12 da. at 8% = $21.59.
6. $736 + $7.66 + $21.59 – $240 = $525.25, 3d principal.
7. Int. on $525.25 for 4 mo. at 8% = $14.01.
8. $525.25 + $14.01 = $539.26, am't due at settlement.

Remark.—The work may be shortened by putting steps (3) and (5) together. Since the payment ($10) is not sufficient to discharge the interest up to that date, we should find the interest on $736 for 8 mo., which is $39.25. The am't is $736 + $39.25 = $775.25. $10 + $240 = $250, am't of the payments. $775.25 − $250 = $525.25, 3d principal in step (6).

363. Business men often settle notes and accounts running for a year or less, upon which partial payments have been made, by the **Mercantile Rule.**

RULE.—*If the time does not exceed one year, subtract the sum of the amounts of the payments to the day of settlement from the amount of the principal to the day of settlement.*

If the time exceeds a year, find the balance due at the end of each year by the rule above stated, and treat it as a new principal.

NOTE.—In applying this rule, since the time is usually short (generally less than a year), it is customary to find *exact interest* (see Sec. **344,** p. 162).

EXAMPLE

A note of $500, dated June 1, 1903, interest at 6%, had the following indorsements:

Aug. 1, 1903, $120; Oct. 1, 1903, $100; Nov. 16, 1903, $25: what was due Dec. 28, 1903?

SOLUTION:

1. Am't of $500 from June 1 to Dec. 28 (210 da.)=....... $517.26.
2. Am't of $120 from Aug. 1 to Dec. 28 (149 da.) =$122.94
3. Am't of $100 from Oct. 1 to Dec. 28 (88 da.) =$101.45
4. Am't of $25 from Nov. 16 to Dec. 28 (42 da.) = $25.17
5. Sum of amounts of payments.............. 249.56
6. Balance due, Dec. 28, 1903................. $267.70

Exercise XLIV

Use the U. S. Rule:

1. A note of $1450, dated Feb. 3, 1902, interest at 5%, was indorsed as follows:

April 3, 1902, $64.10; July 9, 1902, $168.67; Nov. 18, 1903, $20; March 18, 1904, $21: what was due June 14, 1904?

2. $600. PADUCAH, KY., *April* 1, 1902.

On demand, I promise to pay to the order of Pete McGhee, six hundred dollars, for value received, interest at 6%.

JOHN TODD.

Indorsements: July 16, 1902, $63.20; Oct. 16, 1902, $58.05; Dec. 30, 1902, $154.99; May 17, 1903, $9; Sept. 29, 1903, $9. What was due Jan. 1, 1904?

3. $3475. CHICAGO, ILL., *March* 6, 1903.

On demand, I promise to pay to the order of David Sisk, three thousand four hundred seventy-five dollars, for value received, interest at 5%.

BEN SUMMERS.

Indorsements: June 1, 1903, $1247.60; Sept. 10, 1903, $1400. What was due Jan. 31, 1904?

Use the Mercantile Rule:

4. A note of $1750, dated April 5, 1903, drawing 6% interest, has the following indorsements:

May 10, 1903, $190; July 1, 1903, $250; Aug. 5, 1903, $645; Oct. 1, 1903, $372. What was due Dec. 31, 1903?

5. A note of $1000, dated Feb. 1, 1903, with interest at 6%, was indorsed as follows:

March 1, 1903, $200; May 10, 1903, $100; July 1, 1903, $150; Oct. 15, 1903, $40. What was due Jan. 1, 1904?

6. Solve number 5 by the United States Rule also.

VI. EXCHANGE

364. Exchange is the system of sending money from one place to another, or of making payments by written orders, without sending the money.

365. If Jones, of Chicago, wishes to pay a debt in Boston, he can do so by sending the money:

1. By a special messenger. (Usually too expensive.)
2. By unregistered letter. (Unsafe; liable to be lost or stolen.)
3. By registered letter. (Reasonably safe.)
4. By express. (Safe; company is liable for loss.)

In each of the four ways mentioned, *the money is actually transmitted.*

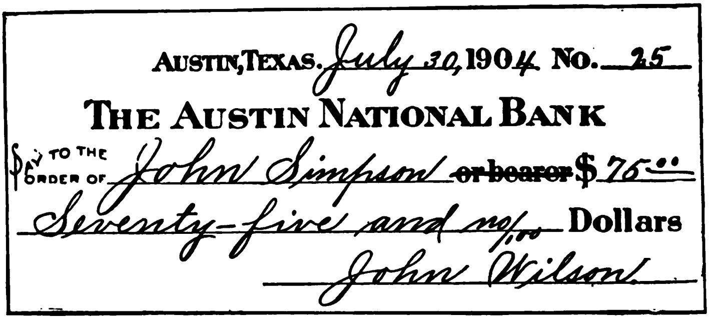
366. Jones may cancel the debt *without actually transmitting the money* by sending:

1. A postal money order. (Reasonably safe.)
2. An express money order. (Safe.)
3. A telegraphic order. (Safe, but expensive.)
4. A check on his home bank where he has money deposited. (Safe, but the creditor may have to pay a small sum to a bank for collecting it.)

5. A draft drawn by the cashier of a bank on some bank in a large city. (Safe.)

NOTE.—Excepting the 4th and the 5th, the above-mentioned methods of paying debts at a distance are so simple that no further explanations need be given here.

367. A **check** is a written order on a bank by a depositor for money standing to his credit.



AUSTIN, TEXAS, July 30, 1904 No. 25

THE AUSTIN NATIONAL BANK

PAY TO THE ORDER OF John Simpson ~~or bearer~~ $75.00

Seventy-five and no/100 Dollars

John Wilson

FORM OF CHECK

NOTE.—John Simpson must indorse the above check before he can draw the money. He may transfer the check to another party, who must also indorse it, as in the case of a promissory note.

If the words "*or bearer*" are used, instead of "*or order*," no indorsement is necessary, and any one in possession of the check may draw the money.

368. A **draft** (also called a *bill of exchange*) is a written order made by one party to another to pay a specified sum to the party named therein.

369. The **parties** to a check or draft:

1. The **drawer** is the one making the order.
2. The **drawee** is the one to whom the order is addressed.

3. The **payee** is the one to be paid the money.

Drafts are either *sight drafts* or *time drafts*.

370. A **sight draft** is payable when presented.

371. A **time draft** is payable a certain time after date, or after sight, allowing 3 days of grace.

372. The **acceptance** of a draft is the promise of the *drawee*, made when the draft is presented, to pay it at maturity. This obligation is acknowledged by writing in red ink across the face of the draft the word "accepted," with the date and the drawee's signature. The draft then becomes an "*acceptance*," and is of the same nature as a promissory note.

373. The **rate of exchange** is the rate above or below par.

1. When the bankers of New Orleans do not have enough money on deposit in New York banks to pay the drafts they are drawing upon New York, they are put to the expense of sending the money, or of paying interest on what they owe. This raises the price of drafts in New Orleans, or exchange on New York is *at a premium*.
2. When the New Orleans bankers have large sums of money deposited in New York banks, often they are willing to sell drafts on New York *at a discount*, to get money to use at home without the expense of having it forwarded by express.
3. Exchange between two cities is generally about the same as the cost of sending the money by express.

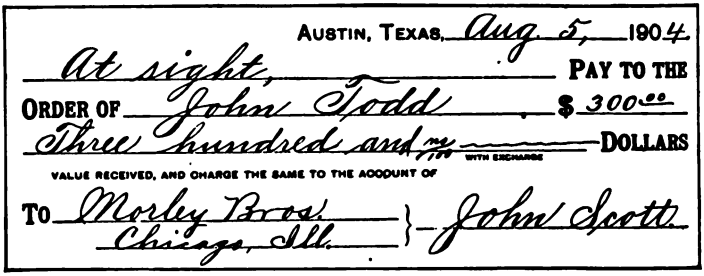
374. Fundamental principles of exchange:

1. In sight drafts, the cost equals the face of the draft plus the premium or minus the discount.

2. In time drafts, the cost equals the face of the draft minus the interest, plus the premium or minus the discount.

(1) DOMESTIC EXCHANGE

375. Domestic or **inland exchange** is exchange between people in the same country.



AUSTIN, TEXAS, Aug. 5, 1904

At sight, PAY TO THE

ORDER OF John Todd $300.00

Three hundred and no/100 DOLLARS

WITH EXCHANGE

VALUE RECEIVED, AND CHARGE THE SAME TO THE ACCOUNT OF

TO Morley Bros.
Chicago, Ill. } John Scott

FORM OF SIGHT DRAFT

NOTE.—A time draft differs from the above in that, instead of "at sight," "......days after sight" or "......days after date" is used. Of course, in time drafts *after sight* means *after acceptance*.

NOTE 2.—This draft was drawn by John Scott, the *drawer*, upon Morley Bros. of Chicago, the *drawee*. The Morley Bros. are hereby ordered to pay John Todd, the *payee*, $300.

Problem: What did John Scott pay for the above draft if the premium was $\frac{1}{4}\%$?

Exercise XLV

1. What will be the cost of a draft on San Francisco for $4300 at $\frac{1}{2}\%$ premium?

2. Find the cost of a draft of $8400 on Chicago, at ½% discount.

3. B paid $1602 for a draft on New York at ⅛% premium. Find the face of the draft.

SOLUTION:

1. Since 100% of face = face,
2. and ⅛% of face = premium,
3. ∴ 100⅛% of face = $1602, the cost,
4. and 1% of face = $\frac{1}{100\frac{1}{8}}$ of $1602 = $16,
5. and 100% of face = 100 × $16 = $1600.

SHORT PROCESS:

$1.00⅛ = cost of $1 face.
$1602 ÷ $1.00⅛ = 1600. ∴ $1600 = face.

What is the face of a draft that costs:

4. $600, premium ¼%? 6. $2000, premium ⅜%?
5. $1300, discount ⅞%? 7. $2000, discount ¼%?

8. Find the cost of a 60-day draft of $1000 on Chicago, if interest is 6%, and the premium on Chicago is ½%.

SOLUTION:

I. 1. Int. on draft for 1 yr. at 6% = 6% of draft.
2. Int. on draft for $\frac{7}{40}$ yr. at 6% = $\frac{21}{20}$% of draft.
3. 100% of draft + ½% of draft − $\frac{21}{20}$% of draft = 99.45% of draft.

II. 1. Since 100% of draft = $1000,
2. ∴ 1% of draft = $\frac{1}{100}$ of $1000 = $10,
3. and 99.45% of draft = 99.45 × $10 = $994.50.

9. What is the cost of a 30-day draft of $1100, premium ⅝%, and interest at 4%?

10. A 90-day draft of $2000 was purchased at a premium

of ¼%. Find the cost of the draft, current rate of interest being 10%.

11. What will be the cost of a draft on El Paso for $600 payable in 60 days, exchange 1½% discount, and interest at 8%?

12. Exchange for $480 cost a merchant $480.60. Find the rate of exchange.

13. Find the face of a 60-day draft purchased for $1500, exchange at 1% discount, interest at 6%.

Solution:

1. Bank discount (*i.e.*, int.) on $1 for 63 days at 6% = $0.0105.
2. Cost at par of each $1 of the draft = $1 − $0.0105 = $0.9895.
3. Discount on each $1 of the draft = $0.01
4. Cost of each $1 of the draft = $0.9795.
5. $1500 ÷ $0.9795 = 1531.39+. ∴ $1531.39+ = the face.

Note.—On time drafts, bankers allow bank discount, *i.e.*, simple interest, for the time specified in the draft. This must be deducted from the cost.

14. Find the face of a draft, payable 30 days after date (without grace), that can be bought for $1000, interest 6%, and exchange ¼% premium.

15. How large a draft, payable in 30 days after sight, can be bought for $352.62, exchange 1½% discount, and interest at 6%?

16. A commission merchant sold $7500 worth of cotton. After deducting his commission of 1½%, he remits the proceeds in a draft purchased at ½% premium. What was the face of the draft?

(2) FOREIGN EXCHANGE

376. Foreign exchange is exchange between people in different countries.

NOTE.—Drafts are expressed in the money of the country in which they are made payable.

377. A **set of exchange** is a draft drawn up in duplicate, named *first* and *second* of exchange, or "Original" and "Duplicate," each copy being valid until *one* has been paid. When one is paid, the other is void. Formerly a draft drawn up in triplicate formed a *set of exchange*. At present, many banks issue only single drafts, but each draft is followed by a *letter of advice*.

SET OF EXCHANGE

£600. NEW YORK, *Feb.* 12, 1903. No. 87.

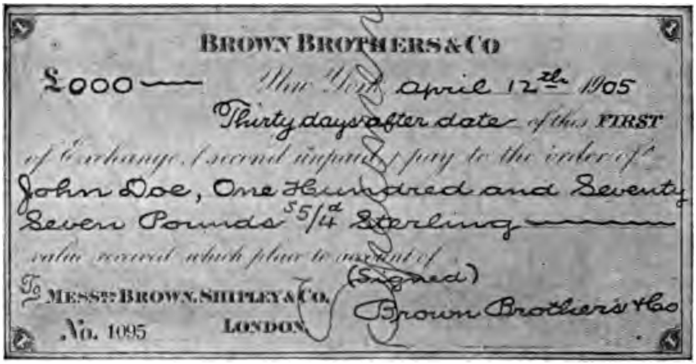
At sight of this first of exchange (second of the same tenor and date unpaid) pay to the order of *Morley Bros., Six hundred pounds sterling,* value received, and charge the same to the account of

RICHARD ROE.

To J. B. DON & Co.,
London, England.

FORM OF THE FIRST DRAFT USED FOR TIME OR SIGHT COMMERCIAL BILLS ISSUED BY MERCHANTS AGAINST SHIPMENTS

NOTE.—The *second draft* is precisely like this, except that the words "*first*" and "*second*" change places.



BROWN BROTHERS & CO

£000 —— New York April 12th 1905

Thirty days after date of this FIRST of Exchange (second unpaid) pay to the order of John Doe, One Hundred and Seventy Seven Pounds $^{s}5/4^{d}$ Sterling ——

value received which place to account of

(Signed)
Brown Brothers & Co

To Messrs. Brown, Shipley & Co.
London

No. 1095

FORM OF DRAFT ISSUED BY ONE BANKER AGAINST ANOTHER BANKER

Note.—In a genuine draft, the sum of £177 5s 4d would be written in the upper left-hand corner of the draft in place of the ciphers in the specimen.

378. The **par of exchange** is the estimated value of the monetary unit of one nation expressed in that of another.

Thus, the *par of exchange* on London is 4.8665; that is, £1 in gold is worth $4.8665 in gold. The *rate* of foreign exchange varies continually, depending upon the demand. When English exchange is quoted at 4.90, it is *above par;* at 4.8665, it is *at par;* at 4.83, it is *below par.*

379. The par of exchange of the monetary units of:

(1) *England.*	(2) *France.*	(3) *Germany.*
£1 = $4.8665.	1 franc = $0.193.	1 mark = $0.238.
		4 marks = $0.952.

380. Foreign exchange is generally quoted in the newspapers as follows, the prices varying:

	Demand.	60 *days.*	90 *days.*
Sterling	4.84 (dollars for £1)	4.79	4.76
Francs	5.20 (francs for $1)	5.22	5.23
Marks	94½ (cts. for 4 marks)	94⅜	93⅞

This means that a draft on England for £1 will cost \$4.84, if payable on demand, or at sight; \$4.79, if payable in 60 days; \$4.76, if payable in 90 days.

EXAMPLES

1. What will a 90-day draft for £50 cost?

SOLUTION:

1. Since £1 cost \$4.76,
2. ∴ £50 cost 50 × \$4.76 = \$238.

2. What will be the cost of a sight draft for 200 marks?

SOLUTION:

1. Since 4 marks cost \$0.945,
2. ∴ 1 mark costs $\frac{1}{4}$ of \$0.945 = \$0.23625,
3. and 200 marks cost 200 × \$0.23625 = \$47.25.

3. Find the cost of a 60-day draft for 1044 francs.

SOLUTION:

1. Since 5.22 francs costs \$1,
2. ∴ 1 franc costs $\frac{1}{5.22}$ of \$1 = $\frac{\$1}{5.22}$,
3. and 1044 francs cost 1044 × $\frac{\$1}{5.22}$ = \$200.

Exercise XLVI

Using the quotations on p. 186, find the cost of:

1. A 60-day draft for £100 10s.
2. A 90-day draft for 6276 francs.
3. A sight draft for 640 marks.

How large a draft can be bought on:

4. Liverpool, payable in 60 days, for \$958?

Table Showing Value of Foreign Coins in United States Gold Dollar

Country.	Standard.	Monetary Unit.	Value in U. S. Gold Dollar.
Argentine Republic	Gold	Peso	$0.965
Austria-Hungary	Gold	Crown	.203
Belgium	Gold	Franc	.193
Bolivia	Silver	Boliviano	.439
Brazil	Gold	Milreis	.546
Canada	Gold	Dollar	1.00
Central America	Silver	Peso	.439
Chile	Gold	Peso	.365
China	Silver	Tael Shanghai	.657
		Haikwan	.732
		Tientsin	.697
		Chefoo	.688
Colombia	Gold	Dollar	1.000
Costa Rica	Gold	Colon	.465
Denmark	Gold	Crown	.268
Ecuador	Gold	Sucre	.487
Egypt	Gold	Pound (100 Piasters)	4.943
Finland	Gold	Mark	.193
France	Gold	Franc	.193
German Empire	Gold	Mark	.238
Great Britain	Gold	Pound Sterling	4.866½
Greece	Gold	Drachma	.193
Haiti	Gold	Gourde	.965
India	Gold	Pound Sterling	4.866½
Italy	Gold	Lira	.193
Japan	Gold	Yen	.498
Liberia	Gold	Dollar	1.00
Mexico	Gold	Dollar	.50
Netherlands	Gold	Florin	.402
Newfoundland	Gold	Dollar	1.014
Norway	Gold	Crown	.268
Panama	Gold	Balboa	1.000
Persia	Silver	Kran	.081
Peru	Gold	Sol	.487
Philippine Islands	Gold	Peso	.500
Portugal	Gold	Milreis	1.08
Russia	Gold	Ruble	.515
Spain	Gold	Peseta	.193
Sweden	Gold	Crown	.268
Switzerland	Gold	Franc	.193
Turkey	Gold	Piaster	.044
Uruguay	Gold	Peso	1.034
Venezuela	Gold	Bolivar	.193

NOTE.—These values may vary slightly at any time.

5. Berlin, payable in 90 days, for $75.10?

6. Paris, payable at sight, for $125?

7. B bought a draft on London for £842. What did it cost if exchange was $4.83\frac{1}{2}$?

8. An agent in Boston having $7536.30 due his employer in England, is directed to remit by draft on Liverpool. What is the face of the draft, if exchange is $4.91\frac{1}{4}$?

MISCELLANEOUS PROBLEMS

1. Divide $870 among A, B, and C, so that B shall get $10 more than C, and A $56 more than B.

2. A dealer bought a carload of produce; the price first declined 12%, then advanced 16%, when he sold. What per cent did he gain?

3. An agent receives $5616 to invest in silk; reserving his commission, 4% of amount invested, how many yards of silk can he buy at $1.50 per yard?

4. If the duty on woolen goods is 20¢ per yard and 40% ad valorem, what must an importer pay on 5000 yards of woolen goods invoiced at 75¢ per yard?

5. The difference between two numbers is 8, and if 7 be added to the less, the sum will be 90% of the greater. What are the numbers?

6. If 16 men can excavate a cellar 48 ft. long, 36 ft. wide, and 8 ft. deep in 12 days of 8 hr. each, in how many days of 10 hr. each can 8 men excavate a cellar 30 ft. long, 27 ft. wide, and 16 ft. deep?

7. Divide $\frac{3}{8}$ by $\frac{5}{7}$; give analysis, and deduce the rule for dividing one fraction by another.

8. Which is better, $1000 with 40% off, or $1000 with discounts 25%, 10%, and 7½% off? How much better?

9. What per cent does B gain who buys goods on 6 mo. time, and sells immediately for cash at cost, money being worth 7%?

10. At what time between 5 and 6 o'clock are the hour and minute hands of a watch exactly together?

11. A man bought a home and gave in payment his note for $9282, with interest at 10%. What sum paid annually will discharge principal and interest at the end of 4 years?

SOLUTION:

1. Each payment must discharge the interest for one year and a *part* of the principal.

2. Each succeeding payment must discharge 10% more of the principal than the preceding. Therefore,

Let 100% = the part of the principal paid 1st yr.
110% = " " " " " " 2d "
121% = " " " " " " 3d "
133.1% = " " " " " " 4th "
464.1% = $9282, the entire principal.

$$1\% = \frac{1}{464.1} \text{ of } \$9282 = \$20.$$

100% = 100 × $20 = $2000, principal paid 1st yr.
$928.20 = interest paid 1st yr.
$2000 + $928.20 = $2928.20, the annual payment.

12. A merchant gives his note, 10% from date, for $2442.04: what sum paid annually will have discharged the whole at the end of 5 yr.?

13. The divisor is 6 times the quotient; ¾ of the quotient is ⅜; find the dividend.

14. What must I pay for 4% stock to get 5% on the investment?

15. Find the number of square yards in a concrete walk 10 feet wide surrounding a rectangular block, the total outside measurement of the walk being 1800 feet.

16. Miss Willard sold her house and lot for \$5000, receiving as much for the house as for the lot; on the lot she gained 4%, and on the house she lost 4%; what was the gain or loss?

17. What is the sum of the first ten terms of an arithmetical series whose 3d term is 12 and 9th term is 42?

18. Insert 3 arithmetical means between 6 and 36.

19. If a piece of silk cost 72 cents per yard, at what price should it be marked so that the merchant may sell it at 10% less than the marked price, and still make 25%? Verify result.

20. A merchant has a false yd. stick, 2 in. too short. What is the value of the cloth he sells for \$72.36?

SOLUTION:

Since the *true result* is such a part of the *false result* as the *false unit* of measure is of the *true unit* of measure, we have the following proportion:

True result : false result : : false unit : true unit.

Substituting in the formula,

The true value : \$72.36 : : 34 : 36.

$$\therefore \text{ the true value} = \frac{34 \times \$72.36}{36} = \$68.34.$$

21. A grocer has a false balance which gives 14½ oz. to the pound: what does he gain by the cheat in selling sugar for \$258.56?

22. An agent remits \$407.15 after deducting 3% brokerage and 25¢ for a draft: find his brokerage.

23. A man owns a farm, mortgaged for ⅔ of its cost; if

he were to sell it for $6000, which would be at an advance of 25% above its cost, how much would remain after paying the mortgage?

24. A has 7 loaves of bread, B 5, and C none. The three eat all the bread, each the same amount; C pays to A and B, 12¢: what should each receive?

25. A borrows from B a sum of money and agrees to pay him by three annual payments of $200 each. If money is worth 5% per annum, compound interest, find the sum borrowed.

26. When a Fahrenheit thermometer stands at 68 degrees, what is the reading of a Centigrade thermometer?

27. Show that $2034_6 = 454_{10}$.

28. A tub of butter seems to weigh 36 pounds in one scale pan and 25 pounds in the other. What is its true weight?

29. A man bought a farm for $4500, and agreed to pay principal and interest in 4 equal annual installments. How much was the annual payment, interest being 6%?

30. If 3 cats catch 3 rats in 3 minutes, how long will it take 100 cats to catch 100 rats?

31. Mary is 24 years old. Mary is twice as old as Ann was when Mary was as old as Ann is now. How old is Ann?

32. If $\frac{1}{4}$ of the time past noon is equal to $\frac{1}{16}$ of the time past midnight, what is the hour?

33. A agrees to work for $2 a day and to forfeit $1 for every day he is idle. At the end of 20 days he receives $25. How many days was he idle?

Examination for County Certificate—Kansas

January 30, 1904.

1. Define annual, simple, and compound interest. Illustrate each by a simple problem.

2. Find the length of a square field containing 2560 acres. Find the length of the longest [straight] line of fence that can be built on this field.

3. $(3\frac{1}{5} + 9.65) \times \frac{3\frac{3}{4}}{3\frac{1}{8}} = ?$

4. A's tax at $6\frac{3}{4}$ mills on the dollar amounted to $270. What was the value of his property, if it was assessed at $\frac{2}{3}$ of its value?

5. A bin is 6 × 8 × 9 feet. Find its capacity in bushels.

6. If I pay $99.75 for a sight draft for $100, what is the rate of discount?

7. How much must be paid for insuring a flouring-mill valued at $24000 for $\frac{5}{6}$ of its value at $1\frac{3}{4}\%$?

Examination for State Certificate—Texas

July 5, 1904.

Answer any eight:

1. Express both as a decimal fraction, and as a common fraction in its lowest terms, each of the following: $\frac{7}{8}\%$; $8\frac{1}{3}\%$; $141\frac{2}{3}\%$.

2. Find the lowest common multiple of 1085 and 651.

3. A merchant fixed a selling price on some cloth to gain 60%, but on account of an incorrect yard stick he made only 52%; how long was the "yard" stick?

4. How many tons of water fall upon an acre in an annual rainfall of 26 inches? (1 cu. ft. of water weighs 62.5 lbs.)

5. What would be the cost of 10 planks, each 18 ft. long, 15 inches wide, two inches thick, at $40 per thousand board-feet?

6. If a bin is 16 ft. long, 6 ft. wide, 4 ft. deep, how many bushels of wheat will it hold? (1 bu. equals 2150.42 cu. in.)

7. How many kilograms of water will a tank hold that is 1.80 m. long, 1.10 m. wide, 75 c.m. deep?

8. If a merchant bought certain goods at trade discounts of "40 and 10 off," and sold them at the same list price at "10 off," what per cent profit did he make?

9. A young man earned $12 a week for 4 years; he saved regularly 60% of his earnings: at the end of four years, how much land could he buy with his savings at $20 an acre, paying "half cash"?

10. Find the "exact interest" on $1000 from Jan. 1, 1904, to date of this examination.

NOTE.—See note 2, 339.

ANSWERS TO PROBLEMS

IN

THE FOUNDATIONS OF HIGHER ARITHMETIC

Exercise I

9. 33511. (Ans. to 3d.) **11.** 30333_5. **13.** 2034431_5.
10. 1130131_6. (Ans. to 4th.) **12.** 24564_7. **14.** 234_5.

Exercise IV

14. 39.026. **15.** $53.39. **16.** 31.8 ft.

Exercise V

4. 26.41. **10.** 11. **11.** 1.

Review, p. 36.

9. 1400 rd. **10.** 7 ft. **11.** June 25, next year. **12.** 11 ft.

Exercise X

14. 49.

Exercise XIII

4. 50.

Review, p. 46.

5. 354 rails. **8.** 100 hr. **9.** 3¾ days. **10.** 500 yd.

Exercise XIV

4. $10.000\frac{8}{9}$.
5. 1.251.
6. $99.35\frac{5}{6}$.
7. 176.4.
9. 676.
10. 5.96.
11. 461.405.
13. 100.
14. 500.
15. 485.46.
16. 1.414.
17. $\frac{41}{333}$.
18. $\frac{5}{24}$.
19. $\frac{13}{144}$.
20. $\frac{12}{13}$.
21. $.7\dot{7}8\dot{3}$.
22. 1.2997.+

Review, p. 52.

11. $\frac{7}{135}$.
12. $\frac{4}{35}$.
13. $11\frac{1}{5}$.
14. $20\frac{5}{12}$.
16. $840.
17. 36 ft.
18. $$779\frac{19}{64}$.
19. $16\frac{2}{3}$ hr.
21. $2\frac{6}{17}$ hr.
22. 20 yr.

Exercise XV

1. 128 lbs. 2. 25 times as loud. 3. 16 times. 4. 16 ft.

Exercise XVI

10. 120 mi.
11. 40 ft.
12. 28.28+ ft.
14. 44.9− in.
15. 225625.
16. $57\frac{1}{2}$ ft.

Exercise XVII

10. 12 ft.
11. $42\frac{1}{4}$ sq. ft.
12. 1350 sq. yd.
13. 6 ft.
14. Base, 33 in. sq.; height, 231 in.
15. 2 in. thick; 4 in. wide; 12 in. long
16. 74.11\frac{1}{4}$.

Exercise XVIII

6. 7 : 3 = $\frac{7}{3}$.
9. 153$\frac{1}{8}$.
10. $\frac{5}{13}$.

Exercise XIX

9. 260 bu.
10. 555 ft.
11. $3.
12. $692.
13. $36.48.
14. 14.4 days.
15. 54 yd.
16. 12 days.
17. $8800.
18. $18.

Exercise XX

1. 30 days.
2. 2 days.
3. 20 men.
4. 384 bbl.
5. 35 men.
6. 8 hr.
7. 4$\frac{1}{2}$ ft.
8. 110 men.
9. 44 days.
10. 60 men.

Exercise XXI

2. C's share, $440.
5. C's share, $180.

Exercise XXII

3. $\frac{2}{3}$.
7. 3280 yd.
9. 113.
11. 8925.
12. 337.68 ft.; 1945.68 ft.
13. 64.32 ft.
14. 1206 yd.
15. 7 sec.
16. 240 ft. from tower.
17. 321.6 ft.
18. 1608 ft.

Exercise XXIII

4. 2.
5. 8.
8. 20 ft.
9. $20971.51.
11. $476.40.
12. $221.025.
13. $100.
14. 3 yr.
15. 6%.

Exercise XXIV

2. $32000.
3. $1375.
4. 390.
5. $1\frac{10}{21}$ in.
6. $280.
7. 4 days.
8. 6 ft.
9. $12\frac{6}{7}$ days.
11. 5 hr.
13. 78, 52, 39.
14. 567.
15. $\frac{9}{44}$ (last fraction).
16. 12884.
17. 1110333_4.
18. 29 ft.
21. $49\frac{1}{11}$ min. past 3 o'clock.
22. $13\frac{11}{13}$ " " " "
23. $27\frac{9}{13}$ " " 6 "
24. $10\frac{10}{11}$ " and $43\frac{7}{11}$ min. past 5 o'clock.
25. $32\frac{8}{11}$ " past 6 P.M.
26. $2\frac{2}{3}$ min.
27. $8\frac{8}{9}$ days.
29. 30 mi.
30. 6 hr.
31. 15 weeks.
33. 90 leaps.
34. 720 mi.
35. 180 ft.

Exercise XXV

26. .00045 oz.
27. 988 sq. ft.
28. 87.5 oz. Troy.
29. 640 sq. ch.
30. 14400 shingles.

Exercise XXVI

7. 47 cups.
8. $15.739+.
10. 792.
11. .177+ days.
12. 49 sacks.
13. 1760 rails.
14. $55.76.
15. $\frac{236}{363}$ acre.
16. 527040 min.
17. 2678400 sec.
18. 36925″
19. $295.80.
20. $238.00, if breadth is not cut; $220.37, if breadth is cut.
21. $400.73.
22. $1.28.

Exercise XXVII

1. 56 min. 40 sec.
2. 6 hr. 53 min. 35 sec.
3. 30 min.
4. 2 hr. 21 min. 50 sec.
5. 106° 27′ 56″.
6. 3 hr. 28 min. 47 sec.
7. 53° 30′ W.
8. 36 min. 12 sec. past 2 o'clock A.M. Sunday.

Exercise XXVIII

1. 44 min. past 11 A.M.
3. 8 min. past 6 P.M.
4. 94° 45′ W.

Exercise XXIX

1. 50° F., 8° R.
2. 20° C., 16° R.
3. 635° F.
4. 832° R., 1904° F.
5. 20° R.

Exercise XXX

5. 78654000 mm.
6. 7201.8 Dm.
7. 40030 mm.
8. 1500000000 sq. m.
9. 54 sq. m.
10. 75000000000 cu. m.
11. 8040 dl.
12. 264+ Dl.
13 184.8 gal.
14. 60000 mg.
15. 1102.31 lb.
17. 7210 g.
18. 2028.25 lb.
20. $9.375.
24. $1.26+per yd.
25. 625 pills.
26. 600 Hl.
27. 4%.
28. $29.918.

Exercise XXXI

5. 80 mi.
9. 10.52+ mi.
11. 169361280 sq. vr.

Exercise XXXII

3. 2880.
4. 3 A. 96 sq. rd.
5. 1.75 A.
6. 3888.
7. 1080 sq. ft.
9. 166½ ft.
15. 5.365 A.
16. 12+ in.
17. 733.04 sq. yd.
18. $10.00.
19. 314.16 sq. ft.
20. 340 sq. ft.
21. 992.15 sq. yd.
22. 32 sq. ft.
23. 12 sq. ft.
24. 7238.2464 sq. in.; 57905.9712 cu. in.
26. 1963.5 sq. ft.
27. 21755.58 sq. ft.
28. 18 cu. ft.
30. 476.4 cu. in.
31. 261.8 cu. in.
32. 236.4+ cu. ft.
33. 5.196+ in.
34. 28 in.
35. 120 lb.
36. A, 21¢, B, 3¢.
39. 1280 sq. ft.
40. 48 lb.

Exercise XXXIII

10. $3.60.
11. $206.25.
12. 14500.
13. $5680.
14. $9500.
16. 2500 bu.
17. $1200.
19. Lost $48.
20. $450.

Exercise XXXIV

1. $250.
3. $60.
4. 80%.
5. Lost 12½%.
6. 44%.
7. $3.75.
8. 37½%.
9. 16⅔%.
10. $3.59⅜.

Exercise XXXV

1. $269.325.
2. $10.
4. $58\frac{46}{63}$%.
5. 5%.
6. $1344.63.
7. 25%.
8. $14.40.
9. $20.
11. $8.13.
12. $600.
13. 2¼%.
14. $2240.

Exercise XXXVI

1. \$135.
2. \$24.40.
3. \$525.
4. 5%.
5. \$4506.341+.
6. \$5141.
7. \$64.37.
8. \$500.
9. 2%.
10. \$53.

Exercise XXXVII

1. $1\frac{1}{2}$%.
2. \$664.70.
3. \$4480.
4. \$53.50.
5. \$567; $1\frac{3}{4}$%.
6. \$2475.
7. 3739.20.
8. \$25200.
9. \$7520.
10. \$1600.

Exercise XXXVIII

1. $\frac{1}{3}$%.
2. $1\frac{1}{3}$%.
3. \$116.34.
4. \$209.
5. \$579.
6. \$513.36.
7. $1\frac{2}{5}$%.
8. \$3400.
9. 35%.
10. \$7996.80.

Exercise XXXIX

1. \$17980.
2. \$2257.50.
3. \$210.
4. \$25500.
5. $3\frac{1}{3}$%.
6. \$5797.50.
8. 75.
9. \$5200.
10. \$2052.
11. \$2450.
12. \$20.
13. \$18550.
14. 438 shares.
15. 48 bonds; \$8 balance.
16. 5%.
17. $1\frac{1}{2}$%; A's part, \$27.
18. U. S. $4\frac{1}{2}$'s, $\frac{1}{2}$%.
19. \$5502.
20. \$360, decrease.

Exercise XL

6. \$2285.098$\frac{2}{3}$.
7. \$6417.80+.
8. 9%.
9. 4%.
10. 1 yr. 4 mo. 15 da.
11. 9 mo.
12. \$360.
13. \$1000.
14. *\$720.*
15. \$2500.
16. \$237.31.
17. \$10.395.
18. \$21.65$\frac{1}{3}$.
19. \$29.24$\frac{68}{73}$.
20. \$6.22$\frac{44}{73}$.
21. Sept. 18.
22. \$346.15.
23. 6%.
24. 12 yr. 6 mo.
25. $8\frac{1}{3}$%.
26. \$4.13.
27. \$101.767.
28. \$70.40.
29. \$16.92.
30. 5%.

Exercise XLI

4. $99.24 gain.
5. $1014.20.
6. $592.53.
7. $1182.66⅔.
8. $645.24.
9. 9%.
10. $4.

Exercise XLII

1. $435.75.
2. $578.60.
3. $7161.
4. $72.625.
5. $350.
6. $1673.40.
7. $1602.

Exercise XLIII

1. $107.75.
2. $140.135.
3. $166.86.
4. $900.407.
5. $2659.95.
6. $2.10.
7. $650.
9. $139.51.
10. $351.50.
11. $96.69.
12. 20%.

Exercise XLIV

1. $1327.41. 2. $351.53. 3. $917.23. 4. $334.56. 5. $545.91.

Exercise XLV

1. $4321.50.
2. $8358.
4. $598.50.
5. $1311.48.
6. $1992.53.
7. $2005.01+.
9. $1102.84.
10. $1953.33.
11. $582.60.
12. ⅛% premium.
14. $1002.50.
15. $360.
16. $7350.75.

Exercise XLVI

1. $481.395.
2. $1200.
3. $151.20.
4. £200.
5. 320 marks.
6. 650 francs.
7. $4071.07.
8. £1533 6s. 6d. 1 far.

MISCELLANEOUS PROBLEMS

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1. A, $\$330\frac{2}{3}$; B, $\$274\frac{2}{3}$; C, $\$264\frac{2}{3}$.
2. $2\frac{2}{25}\%$.
3. 3600 yd.
4. $2500.
5. 10 and 2.
6. 18 days.
9. $3\frac{1}{2}\%$.
10. 27 min. $16\frac{4}{11}$ sec., past 5 o'clock.
12. $644.204.
13. $1\frac{1}{2}$.
14. 80.
15. $1955\frac{5}{9}$ sq. yd.
16. $8.01 loss.
17. 245.
21. $24.24.
22. $12.60.
23. $2800.
24. A, 9¢; B, 3¢.
25. $544.64.
26. 20° C.
28. 30 lb.
29. $1298.66.
30. 3 min.
31. 18 yr.
32. 4 P. M.
33. 5 days.

Examination for County Certificate—Kansas

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2. Length of field, 2 mi.;
" " fence, $2\sqrt{2}$ mi.
3. 15.42.
4. $60000.
5. $347\frac{1}{7}$ bu.
6. $\frac{1}{4}\%$.
7. $350.

Examination for State Certificate—Texas

Page 192

2. 3255.
3. $37\frac{17}{19}$ in.
4. 2949.375 T.
5. $18.
6. $308.56+.
7. 1485 Kg.
8. $66\frac{2}{3}\%$.
9. 149.76 A.
10. $30.49+.

To avoid fine, this book should be returned on
or before the date last stamped below

10M—6.40

5/4

Tx
511.6
S622

Sisk, B.F.
Higher arithmetic.

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NAME	DATE	NAME	DATE
Dr. T. B. Kinney A J Hall	[illegible]		

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